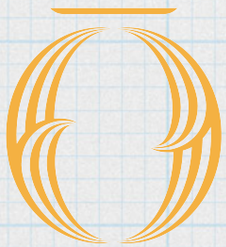
Big bang dynamics of cosmological fluids

Crete Gravity Days, University of Crete, Greece

Florian Beyer, University of Otago, New Zealand

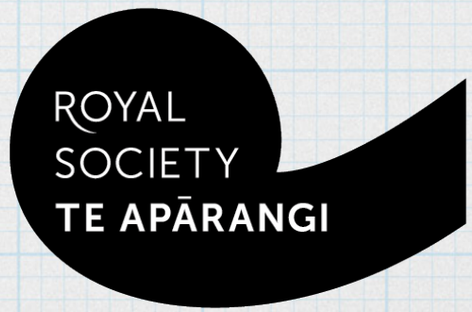
November 4, 2026



University of Otago

ŌTĀKOU WHAKAIHU WAKA





A very brief history

A very brief history

- 1920/30: Friedmann-Lemaitre-Robertson-Walker (FLRW) models: spatially homogeneous and isotropic.

A very brief history

- 1920/30: Friedmann-Lemaitre-Robertson-Walker (FLRW) models: spatially homogeneous and isotropic.
- 1960/70: Penrose/Hawking singularity theorems. Penrose's cosmic censorship conjectures.

A very brief history

- 1920/30: Friedmann-Lemaitre-Robertson-Walker (FLRW) models: spatially homogeneous and isotropic.
- 1960/70: Penrose/Hawking singularity theorems. Penrose's cosmic censorship conjectures.
- 1960/70: Heuristic investigations of "general singularities" by Belinski, Khalatnikov and Lifshitz (BKL).

A very brief history

- 1920/30: Friedmann-Lemaitre-Robertson-Walker (FLRW) models: spatially homogeneous and isotropic.
- 1960/70: Penrose/Hawking singularity theorems. Penrose's cosmic censorship conjectures.
- 1960/70: Heuristic investigations of "general singularities" by Belinski, Khalatnikov and Lifshitz (BKL).
- 1970-2010: Investigate solutions with symmetries (Eardley, Sachs, Moncrief, Isenberg, Berger, Garfinkle, Rendall, Andersson, Choquet-Bruhat, Ringström, …).

Brief history (cont'd …)

Result (Rodnianski, Speck, 2018): The FLRW solution of the Einstein-scalar field equations is *non-linearly stable towards the big bang*.

Brief history (cont'd …)

Result (Rodnianski, Speck, 2018): The FLRW solution of the Einstein-scalar field equations is *non-linearly stable towards the big bang*.

Extensions and refinements: Fournodavlos, Luk, Ringström, Groeninger, Petersen, Fajman, Oliynyk, B., …

Flat FLRW solutions

Spatially homogeneous and isotropic solutions with *big bang singularities* at $t = 0$:

$$g = -dt^2 + a^2(t) \sum_{I=1}^{3} dx_I^2$$

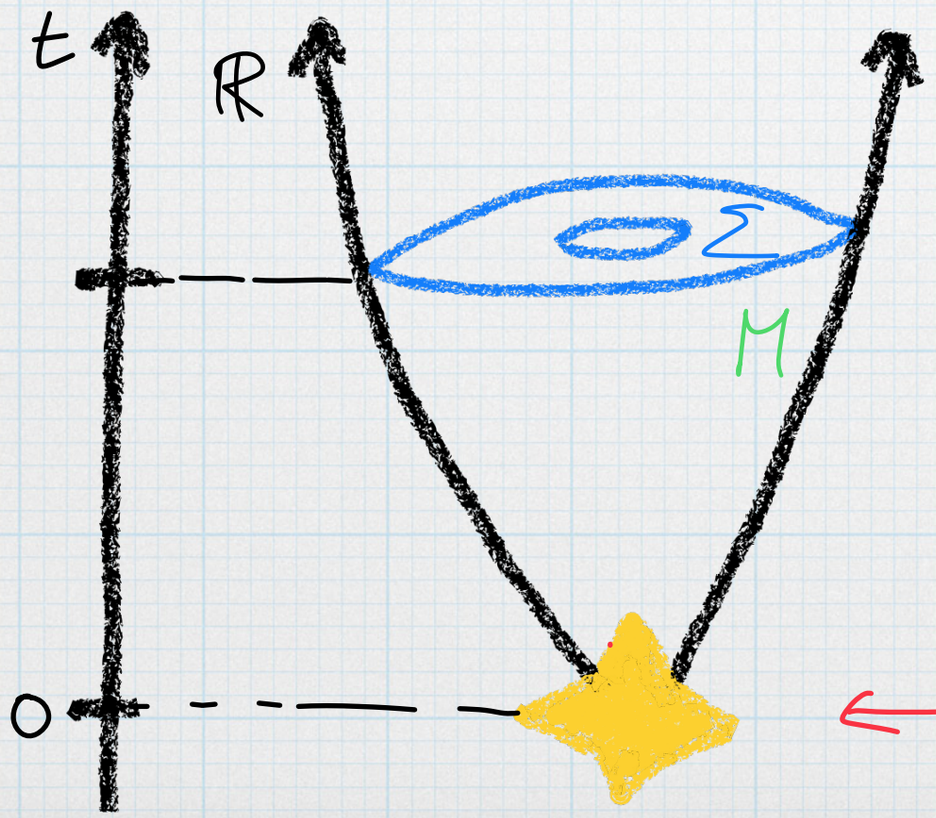
Flat FLRW solutions

Spatially homogeneous and isotropic solutions with *big bang singularities* at $t = 0$:

$$g = -dt^2 + a^2(t) \sum_{I=1}^{3} dx_I^2$$

↳ cosmic scale factor:
$a(t) \sim t^p$, $p > 0$

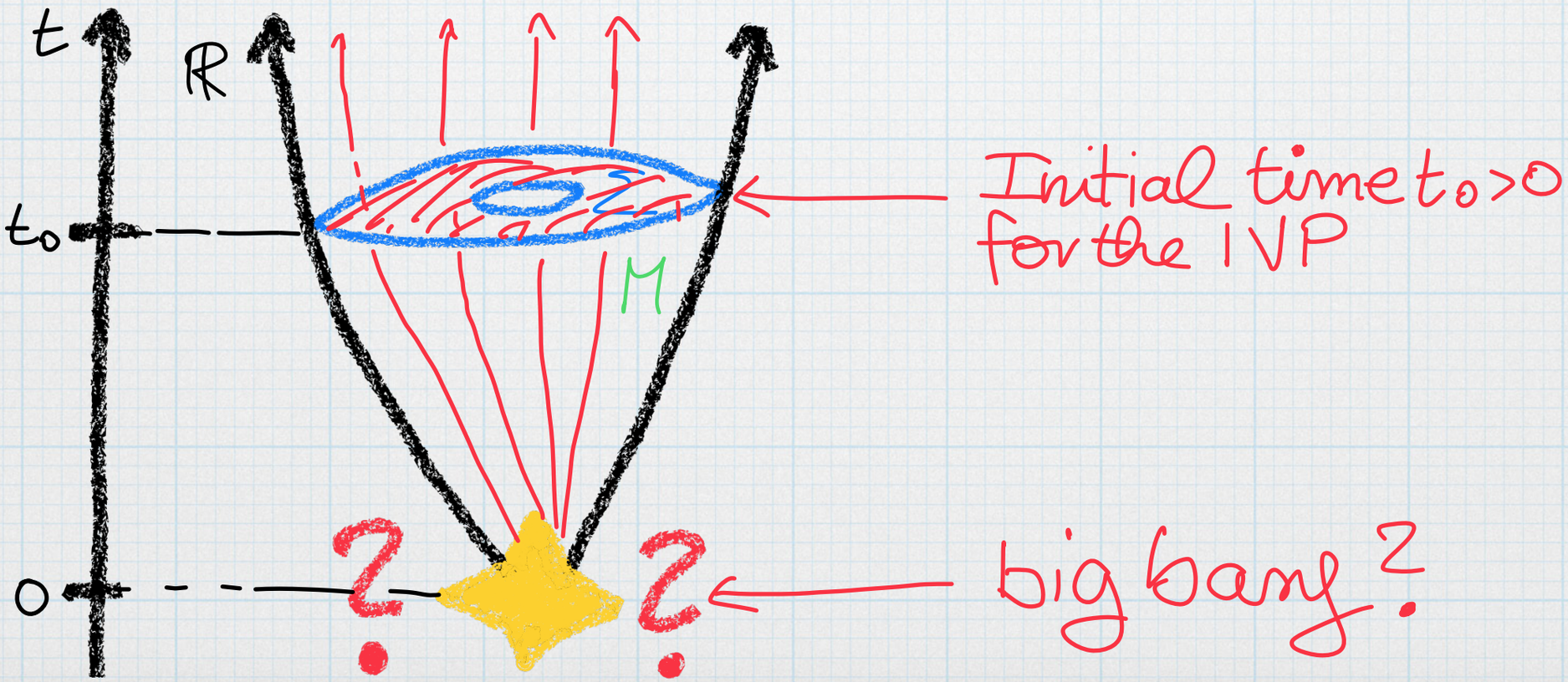
time $t > 0$



big bang

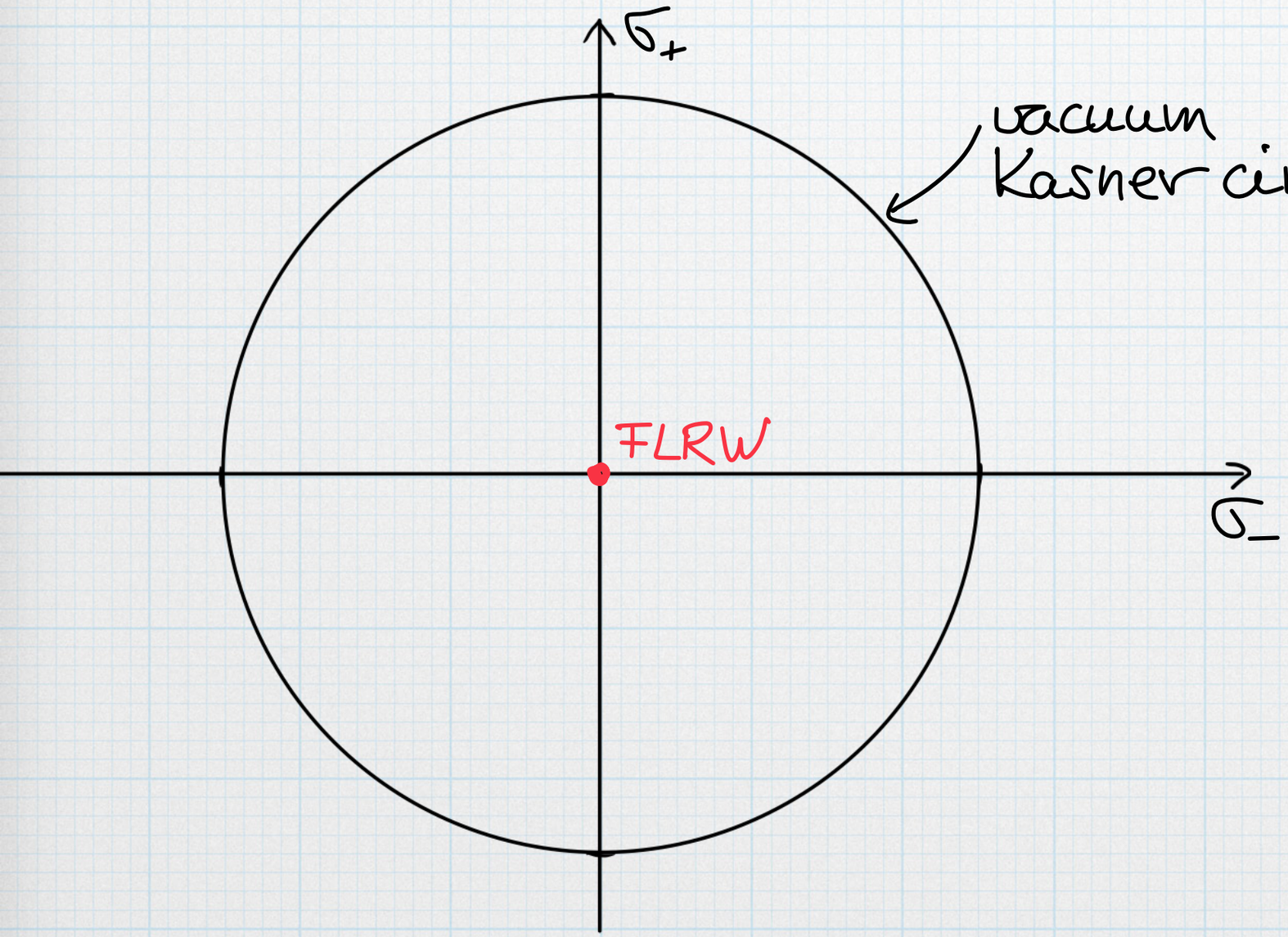
Non-linear perturbations of flat FLRW solutions

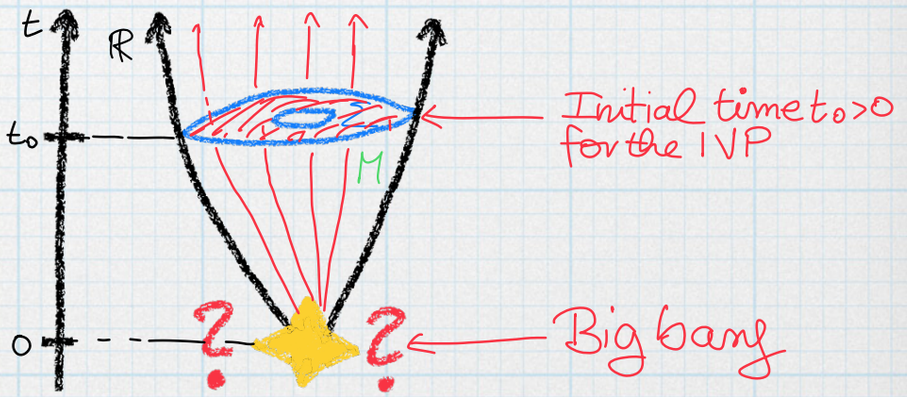
Question: Given arbitrary initial data *close to* the initial data of the FLRW solution, does the corresponding solution of the IVP stay *close to* the FLRW solution globally in time (especially near the *big bang*)?



Stability: Expectations

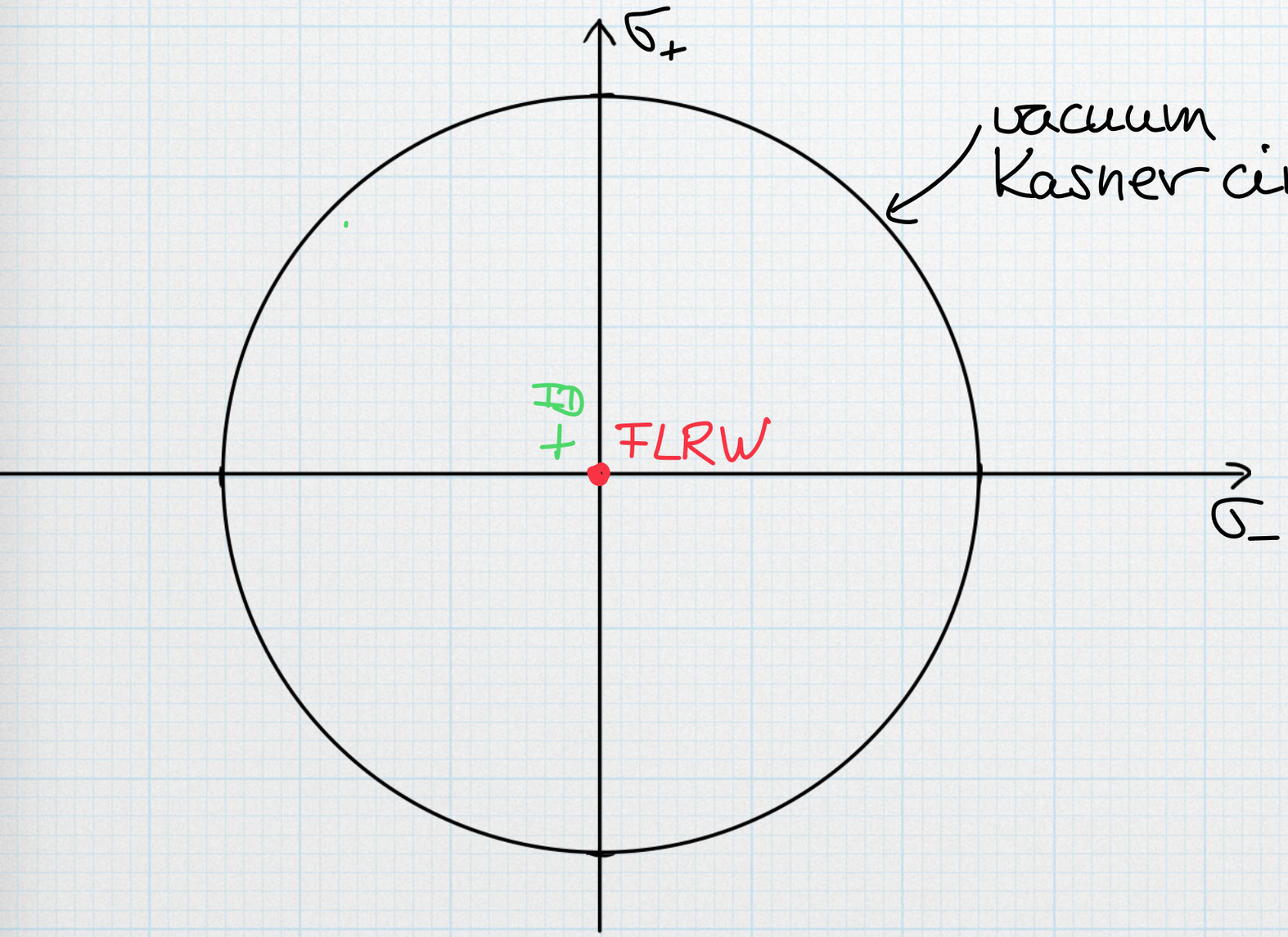
Spatially homogeneous solutions without matter

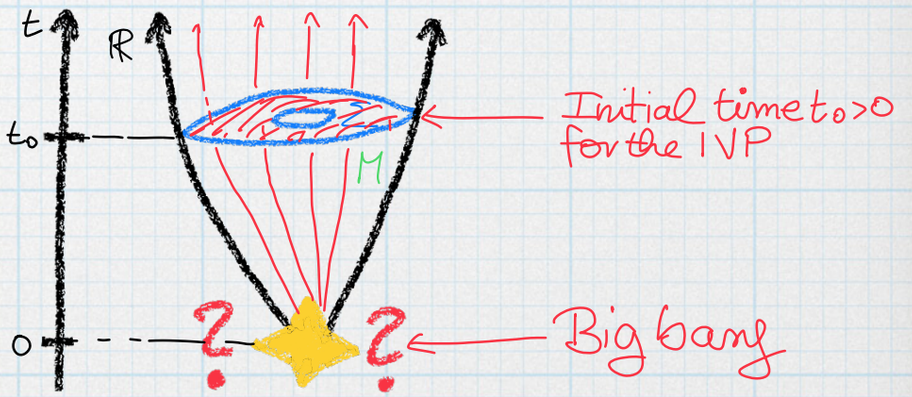




Stability: Expectations

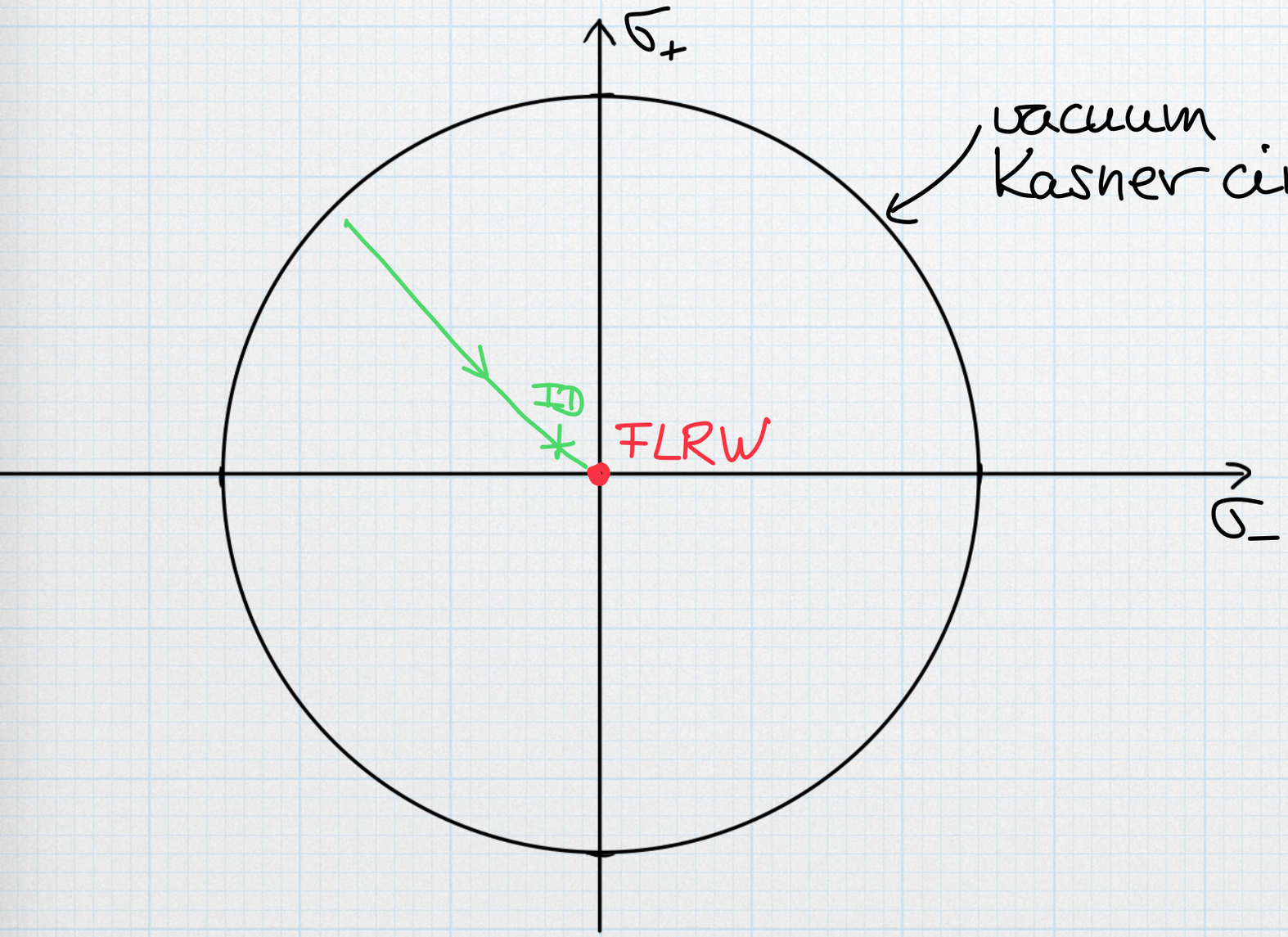
Spatially homogeneous solutions without matter

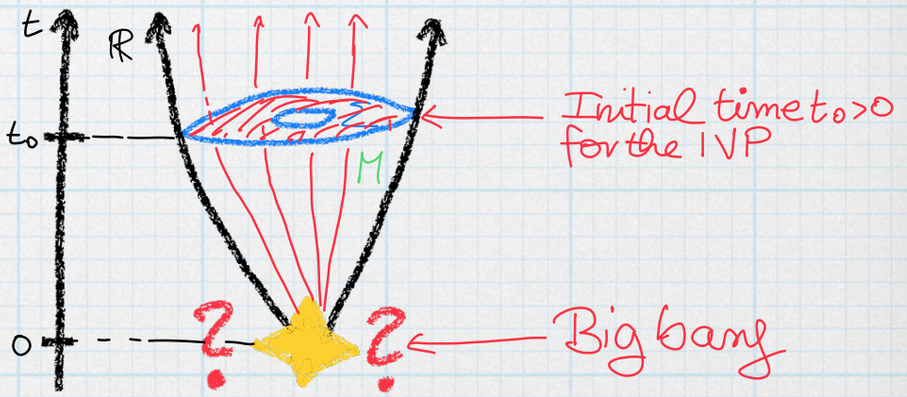




Stability: Expectations

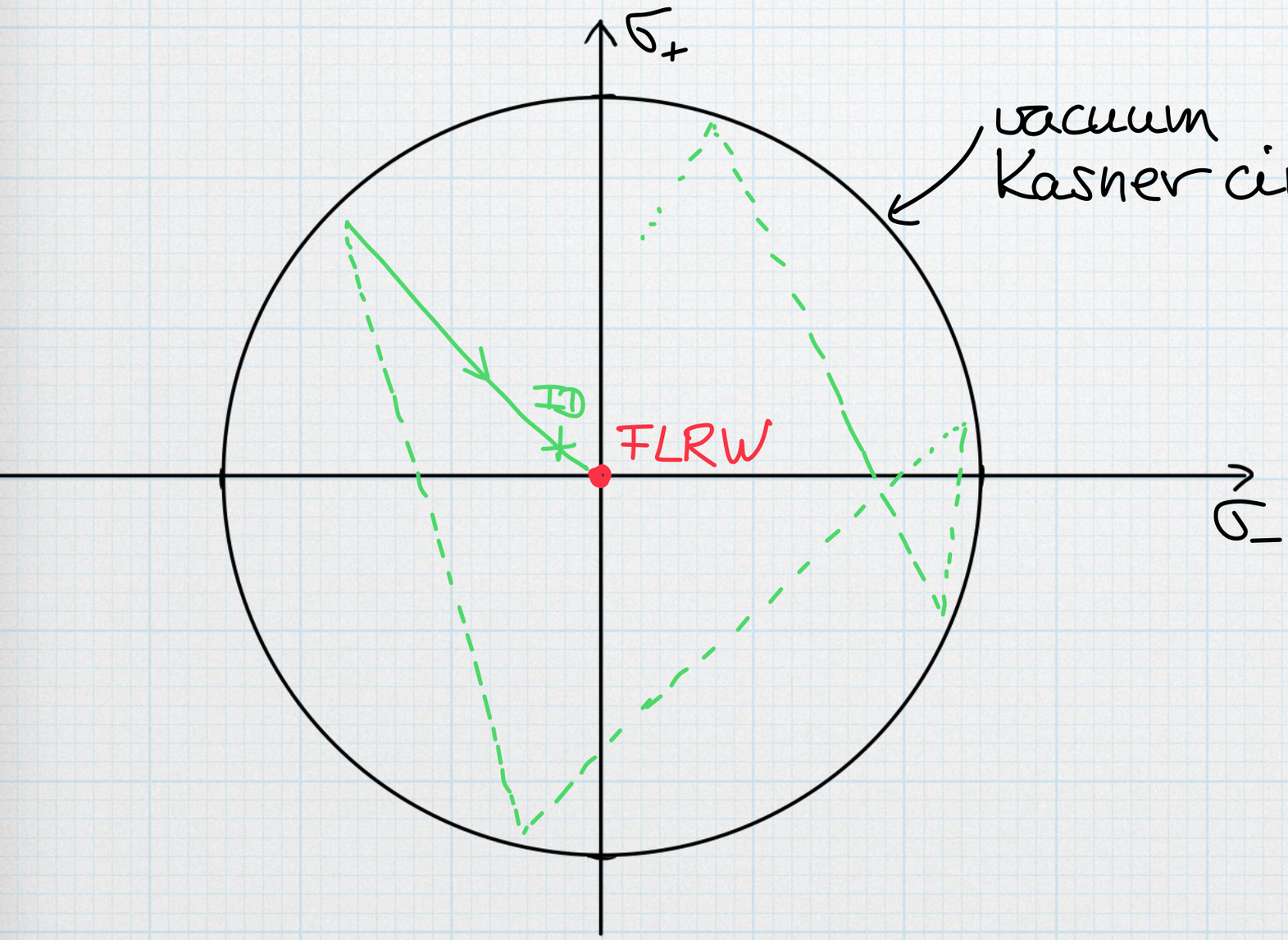
Spatially homogeneous solutions without matter

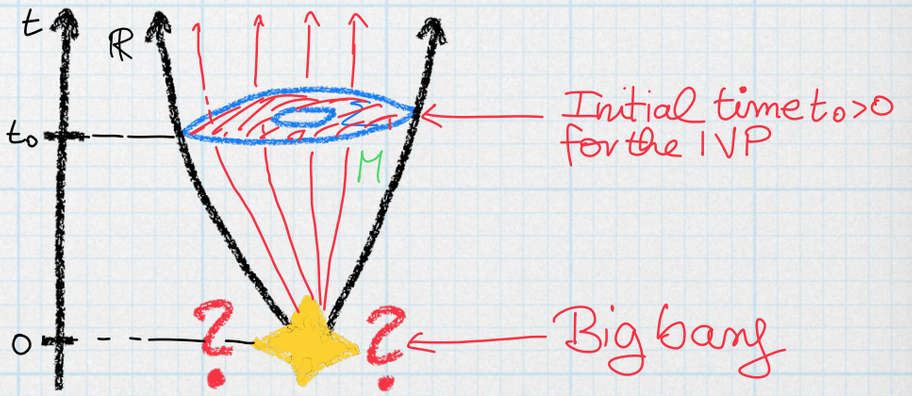




Stability: Expectations

Spatially homogeneous solutions without matter

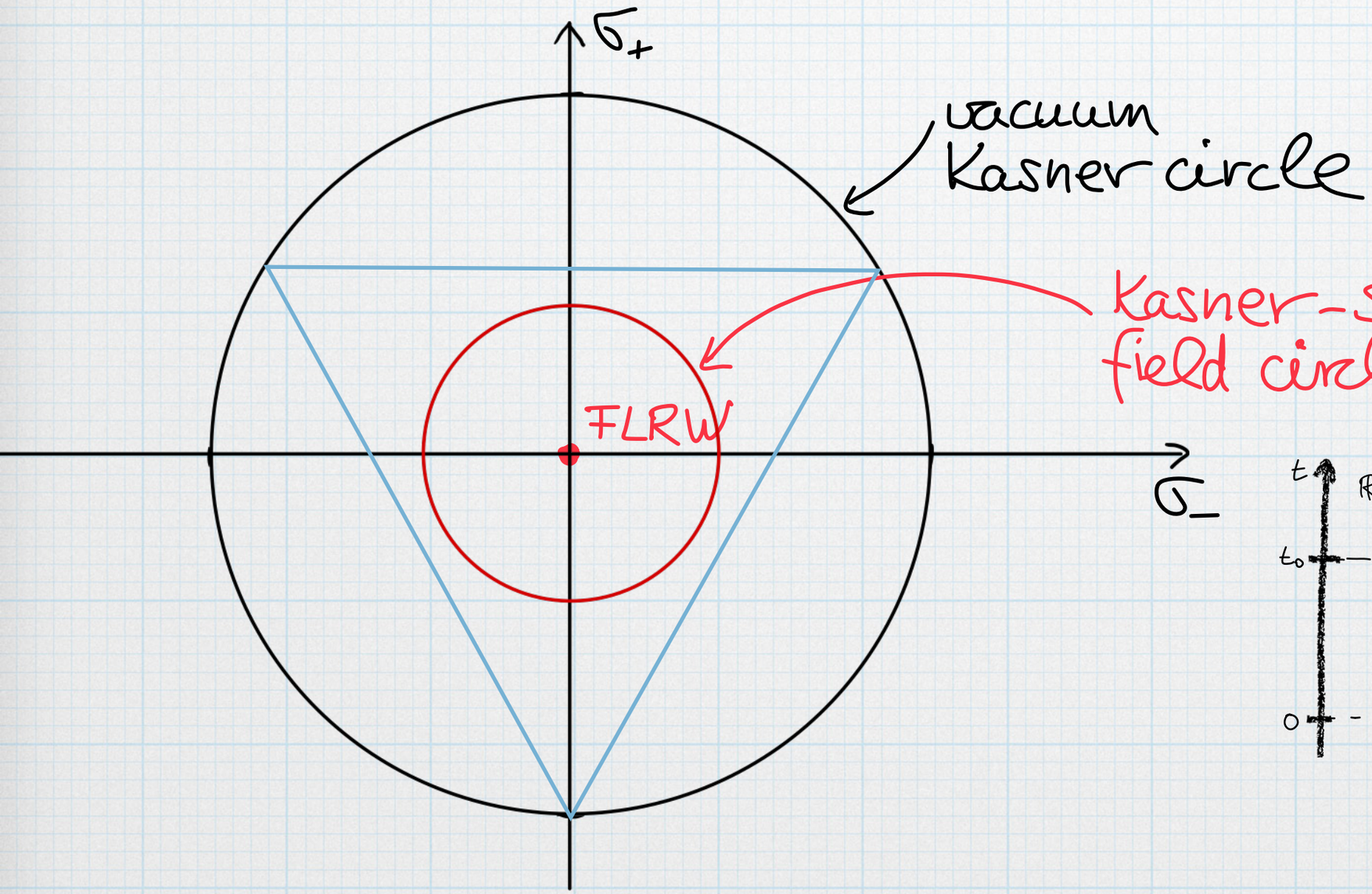


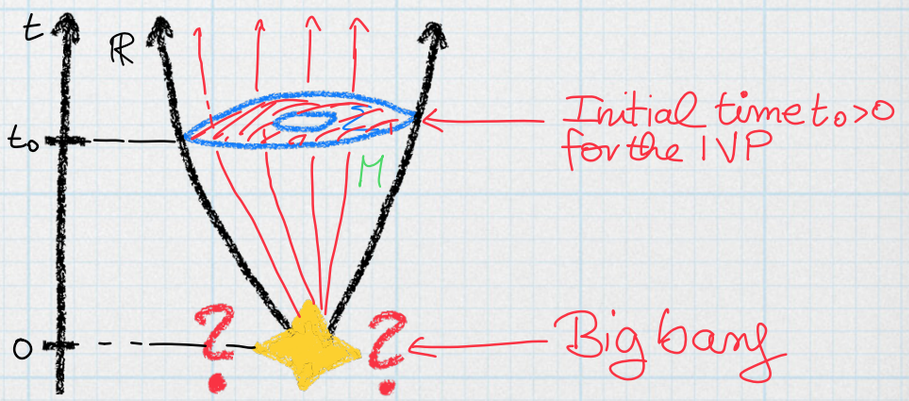


Mixmaster dynamics (BKL, Misner)

Stability: Expectations

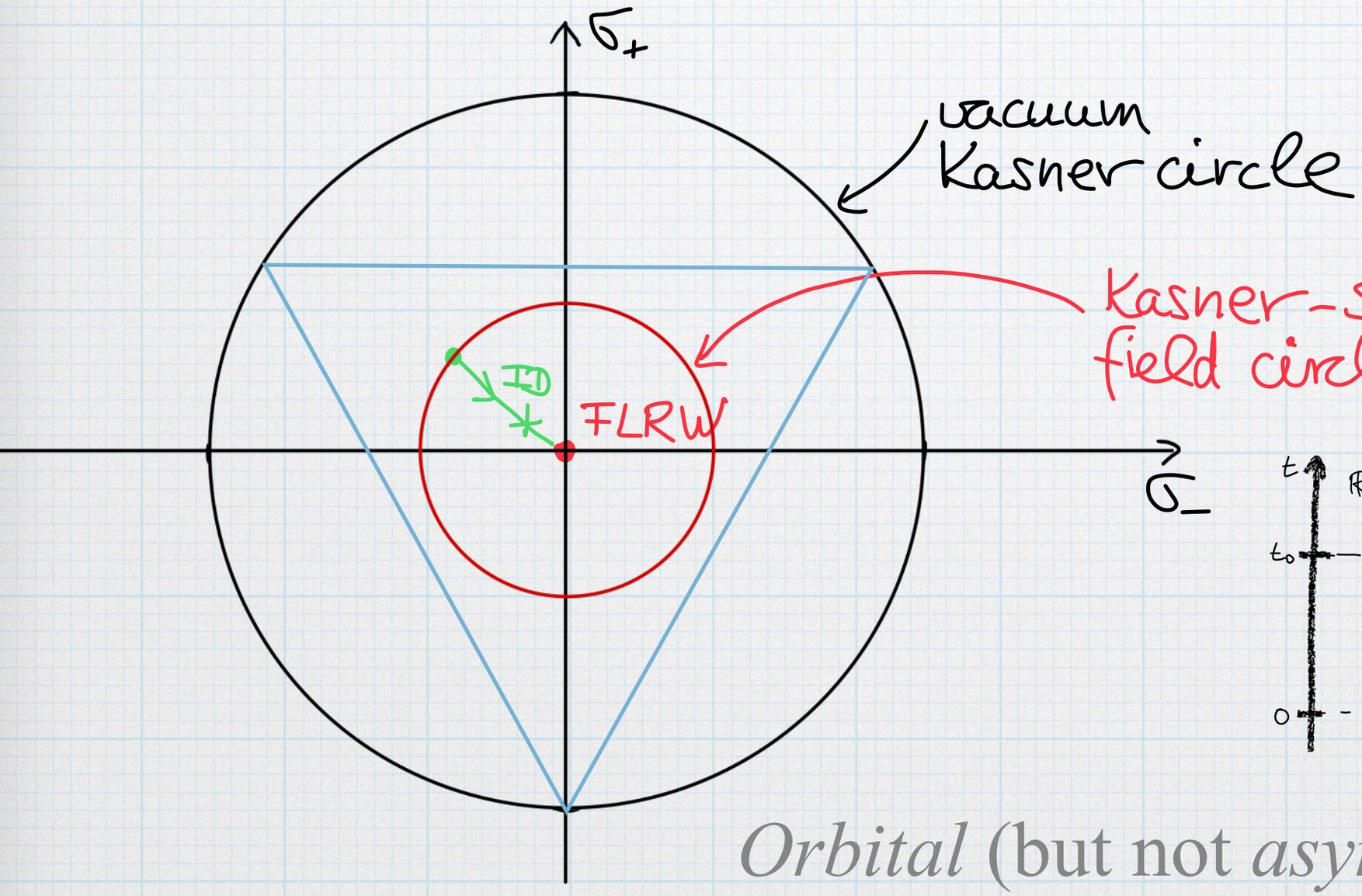
Spatially homogeneous case *with a scalar field*

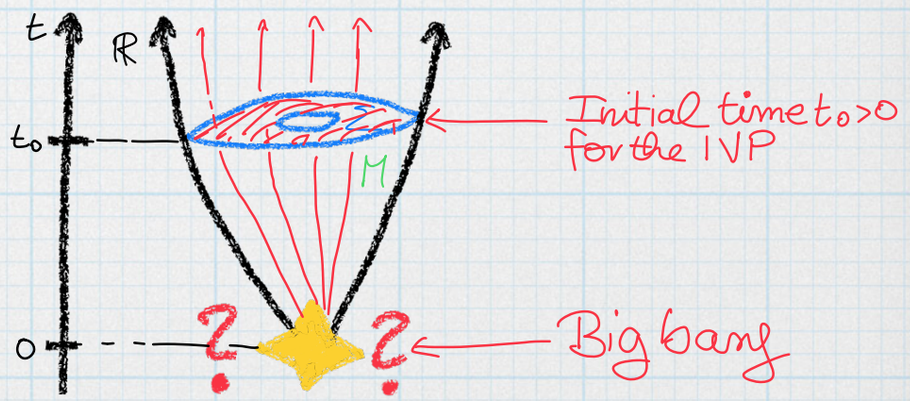




Stability: Expectations

Spatially homogeneous case *with a scalar field*





Orbital (but not *asymptotic*) *stability*

Kasner(-scalar field) spacetimes

Kasner(-scalar field) spacetimes

Spatially homogeneous solutions of the Einstein-scalar field equations:

$$g = -dt^2 + \sum_{I=1}^{3} t^{2p_I} dx_I^2, \quad \phi = A \log t,$$

for *Kasner exponents* $p_1, p_2, p_3 \in \mathbb{R}$ and $A \in \left[-\sqrt{\frac{2}{3}}, \sqrt{\frac{2}{3}}\right]$ such that the *Kasner relations* hold

$$\sum_{i=1}^{3} p_i = 1, \quad \sum_{i=1}^{3} p_i^2 = 1 - A^2.$$

Kasner(-scalar field) spacetimes

Spatially homogeneous solutions of the Einstein-scalar field equations: $$g = -dt^2 + \sum_{I=1}^{3} t^{2p_I} dx_I^2, \quad \phi = A \log t,$$

for *Kasner exponents* $p_1, p_2, p_3 \in \mathbb{R}$ and $A \in \left[-\sqrt{\frac{2}{3}}, \sqrt{\frac{2}{3}}\right]$ such that the *Kasner relations* hold

$$\sum_{i=1}^{3} p_i = 1, \quad \sum_{i=1}^{3} p_i^2 = 1 - A^2.$$

Special cases:

- Vacuum $A = 0$ (*Kasner-vacuum solutions*).

Kasner(-scalar field) spacetimes

Spatially homogeneous solutions of the Einstein-scalar field equations: $$g = -dt^2 + \sum_{I=1}^{3} t^{2p_I} dx_I^2, \quad \phi = A \log t,$$

for *Kasner exponents* $p_1, p_2, p_3 \in \mathbb{R}$ and $A \in \left[-\sqrt{\frac{2}{3}}, \sqrt{\frac{2}{3}}\right]$

such that the *Kasner relations* hold

$$\sum_{i=1}^{3} p_i = 1, \quad \sum_{i=1}^{3} p_i^2 = 1 - A^2.$$

Special cases:

- Vacuum $A = 0$ (*Kasner-vacuum solutions*).
- $A = \pm\sqrt{2/3}$: $p_1 = p_2 = p_3 = 1/3$ (*flat FLRW*).

Relativistic fluid models

Relativistic fluid models

Cosmological matter is typically modelled as a *perfect fluid* with linear equation of state $P = c_s^2 \rho$ with parameter c_s^2 (*speed of sound*). Assume $0 \leq c_s^2 < 1$.

Relativistic fluid models

Cosmological matter is typically modelled as a *perfect fluid* with linear equation of state $P = c_s^2 \rho$ with parameter c_s^2 (*speed of sound*). Assume $0 \leq c_s^2 < 1$.

Different kinds of matter give rise to different values of c_s^2. Especially for the early universe:

$$c_s^2 = \frac{1}{3}, \quad \text{radiation.}$$

Relativistic fluid models

Cosmological matter is typically modelled as a *perfect fluid* with linear equation of state $P = c_s^2 \rho$ with parameter c_s^2 (*speed of sound*). Assume $0 \leq c_s^2 < 1$.

Different kinds of matter give rise to different values of c_s^2. Especially for the early universe:

$$c_s^2 = \frac{1}{3}, \quad \text{radiation.}$$

Energy-momentum tensor:

$$T_{ij} = (\rho + P) u_i u_j + P g_{ij} \,.$$

Euler equations:

$$\nabla^i T_{ij} = 0.$$

Comovingness and tilt

Comovingness and tilt

Decompose the fluid 4-velocity

$$u^i = \Gamma\left(n^i + \nu^i\right), \quad \nu_i n^i = 0, \quad \Gamma = \left(1 - |\nu|^2\right)^{-1/2}.$$

Comovingness and tilt

Decompose the fluid 4-velocity

$$u^i = \Gamma\left(n^i + \nu^i\right), \quad \nu_i n^i = 0, \quad \Gamma = \left(1 - |\nu|^2\right)^{-1/2}.$$

We call ν^i *tilt vector* and Γ the *Lorentz tilt factor*.

- *Comoving/non-tilted* matter: $\nu^i = 0, \quad \Gamma = 1.$

Comovingness and tilt

Decompose the fluid 4-velocity

$$u^i = \Gamma\left(n^i + \nu^i\right), \quad \nu_i n^i = 0, \quad \Gamma = \left(1 - |\nu|^2\right)^{-1/2}.$$

We call ν^i *tilt vector* and Γ the *Lorentz tilt factor*.

- *Comoving/non-tilted* matter: $\nu^i = 0, \quad \Gamma = 1.$
- *Extremely-tilted* matter: $|\nu| \to 1, \quad \Gamma \to \infty$.

Comovingness and tilt

Decompose the fluid 4-velocity

$$u^i = \Gamma\left(n^i + \nu^i\right), \quad \nu_i n^i = 0, \quad \Gamma = \left(1 - |\nu|^2\right)^{-1/2}.$$

We call ν^i *tilt vector* and Γ the *Lorentz tilt factor*.

- *Comoving/non-tilted* matter: $\nu^i = 0, \quad \Gamma = 1.$
- *Extremely-tilted* matter: $|\nu| \to 1, \quad \Gamma \to \infty$.

On Kasner backgrounds, the spatially homogeneous Euler equations imply

$$\left(\Gamma^2 - 1\right)\Gamma^{-2c_s^2} = b_* t^{2(c_s^2 - p)}, \quad b_* \geq 0.$$

Spatially homogeneous fluids on Kasner backgrounds

Spatially homogeneous fluids on Kasner backgrounds

Euler equations imply

$$\left(\Gamma^2 - 1\right)\Gamma^{-2c_s^2} = b_* t^{2(c_s^2 - p)}, \quad b_* \geq 0.$$

Asymptotic regimes:

- **Asymptotically non-tilted**:

  $\Gamma \to 1$ for $t \to 0$ if $c_s^2 > p$ (or $b_* = 0$).

Spatially homogeneous fluids on Kasner backgrounds

Euler equations imply

$$\left(\Gamma^2 - 1\right)\Gamma^{-2c_s^2} = b_* t^{2(c_s^2 - p)}, \quad b_* \geq 0.$$

Asymptotic regimes:

- **Asymptotically non-tilted**:

  $\Gamma \to 1$ for $t \to 0$ if $c_s^2 > p$ (or $b_* = 0$).

- **Asymptotically extremely-tilted**:

  1. $\Gamma \to \infty$ for $t \to 0$ if $c_s^2 < p$ and $b_* \neq 0$.

Spatially homogeneous fluids on Kasner backgrounds

Euler equations imply

$$\left(\Gamma^2 - 1\right)\Gamma^{-2c_s^2} = b_* t^{2(c_s^2 - p)}, \quad b_* \geq 0.$$

Asymptotic regimes:

- **Asymptotically non-tilted**:
  $\Gamma \to 1$ for $t \to 0$ if $c_s^2 > p$ (or $b_* = 0$).
- **Asymptotically extremely-tilted**:
  1. $\Gamma \to \infty$ for $t \to 0$ if $c_s^2 < p$ and $b_* \neq 0$.
  2. The tilt vector ν^i aligns with the direction of the *largest Kasner exponent*.

Spatially homogeneous fluids on Kasner backgrounds

Euler equations imply

$$\left(\Gamma^2 - 1\right)\Gamma^{-2c_s^2} = b_* t^{2(c_s^2 - p)}, \quad b_* \geq 0.$$

Asymptotic regimes:

- **Asymptotically non-tilted**:
  $\Gamma \to 1$ for $t \to 0$ if $c_s^2 > p$ (or $b_* = 0$).
- **Asymptotically extremely-tilted**:
  1. $\Gamma \to \infty$ for $t \to 0$ if $c_s^2 < p$ and $b_* \neq 0$.
  2. The tilt vector ν^i aligns with the direction of the *largest Kasner exponent*.
- **Critical case**: $\Gamma = const$ if $c_s^2 = p$.

Asymptotically non-tilted case

Result 1 (B., Oliynyk, 2024): Solutions of the Euler equations on fixed Kasner-scalar field backgrounds with $1 > c_s^2 > \max_{i=1,2,3} p_i$ are *asymptotically non-tilted* towards the big bang, i.e., there are functions w^1, w^2, w^3 that converge at $t = 0$ such that

$$\rho = O(t^{-(1+c_s^2)}), \quad \nu^i = \sum_{I=1}^{3} t^{c_s^2 - p_I} w^I e_I^i, \quad \Gamma = \left(1 - |\nu|^2\right)^{-1/2} \to 1.$$

Asymptotically non-tilted case

Result 2 (B., Oliynyk, 2024): Suppose $c_s^2 > 1/3$. The FLRW solution of the Einstein-scalar field-Euler equations is *non-linearly stable towards the big bang* and the perturbed solutions are *asymptotically non-tilted*.

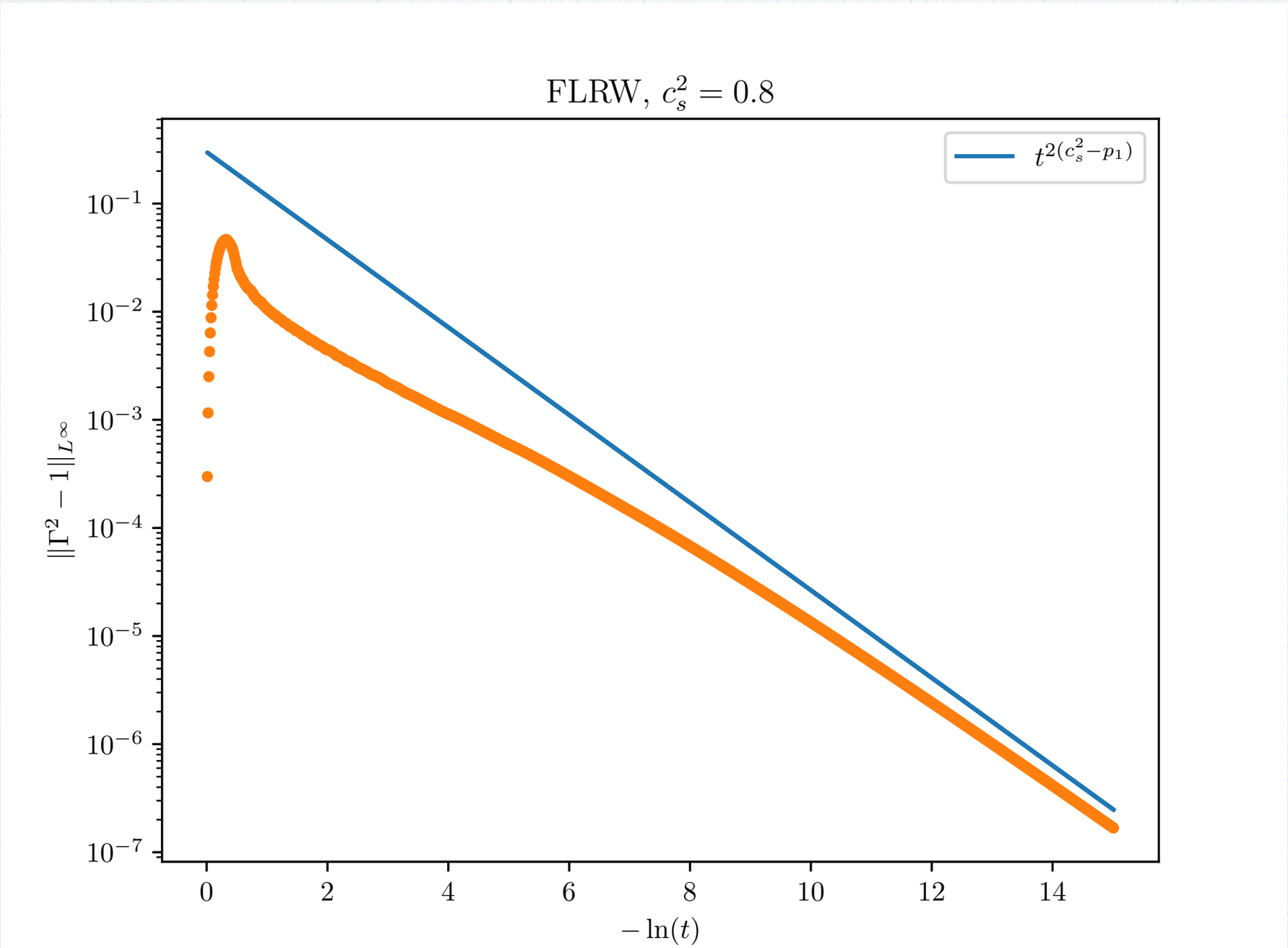
Asymptotically extremely-tilted case

Result 3 (B., 2026): Suppose

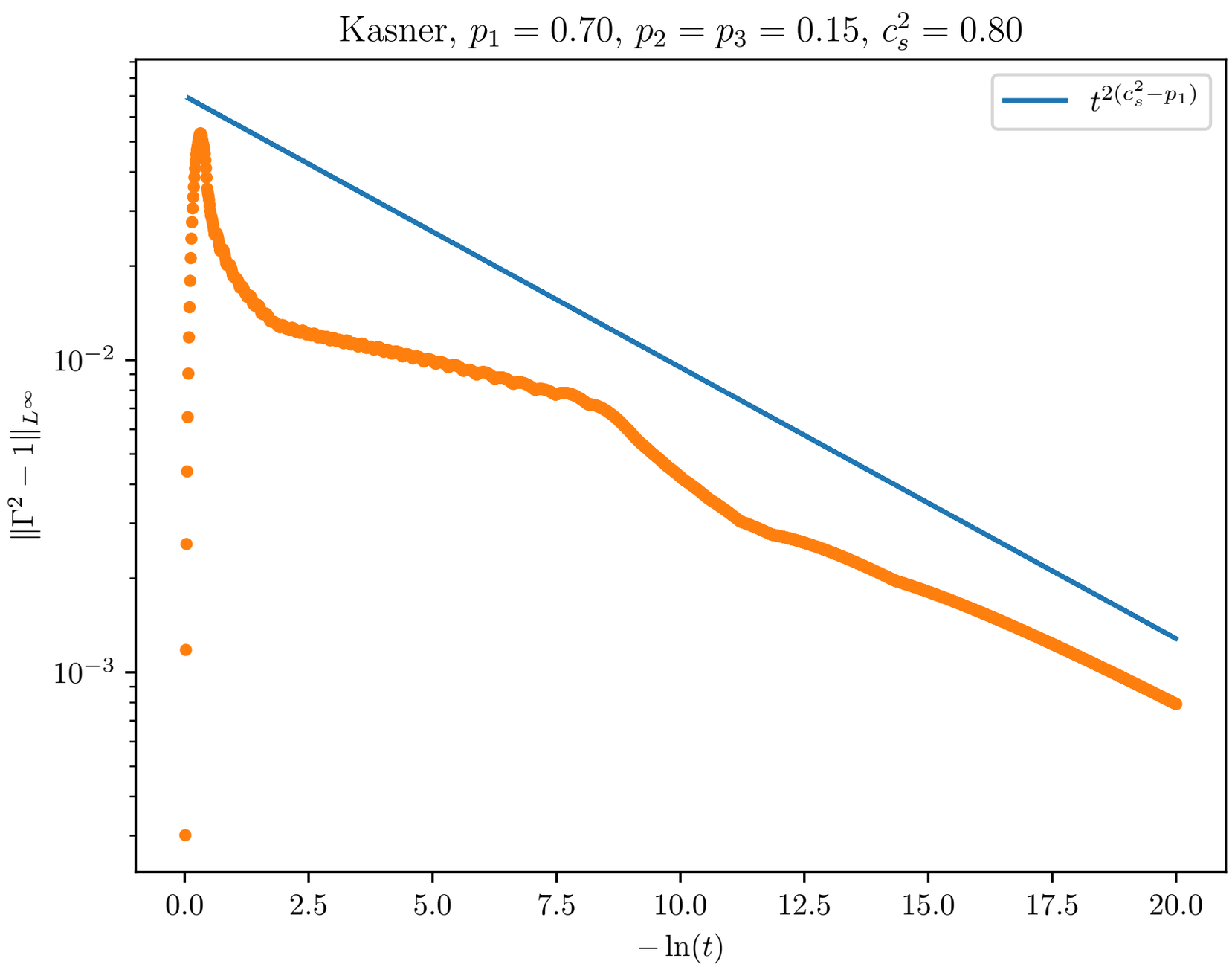
$$1 > p_1 > c_s^2 > c_*^2, \quad p_1 \geq p_2 \geq p_3 \,.$$

Then all* solutions of the Euler equations on a Kasner-type background* launched from a spacelike hypersurface with sufficiently large mean curvature H_0 is asymptotically extremely tilted near the big bang. The *tilt vector* converges to a unit eigenvector of the Weingarten map corresponding to the largest Kasner exponent p_1 and $\Gamma \to \infty$, $\rho = O\left(t^{-(1+c_s^2)\frac{1-p_1}{1-c_s^2}}\right)$.

Asymptotically non-tilted: isotropic



Asymptotically non-tilted: anisotropic



Asymptotically extremely-tilted: anisotropic

