Heraklion, 25 April 2024

Quantum Mechanics, Entanglement and Gravity

Elias Kiritsis

Quantum Entanglement 1.0

• Consider a pair of particles with spin 1/2, produced from a source in a spin-singlet state:

$$|\psi\rangle = \Big(|\downarrow\rangle_1 \otimes |\uparrow\rangle_2 - |\uparrow\rangle_1 \otimes |\downarrow\rangle_2 \Big)$$

• This is the simplest version of an entangled state of two particles (where we focus only on the spin_z degree of freedom)

• The particles move in opposite directions, and we pose two detectors far away that can measure the spin_z of the particles.

• If one detector measures the $\text{spin}_z = \frac{1}{2}$ for particle No 2, then he automatically knows that particle No 1 is in a $\text{spin}_z = -\frac{1}{2}$ state.

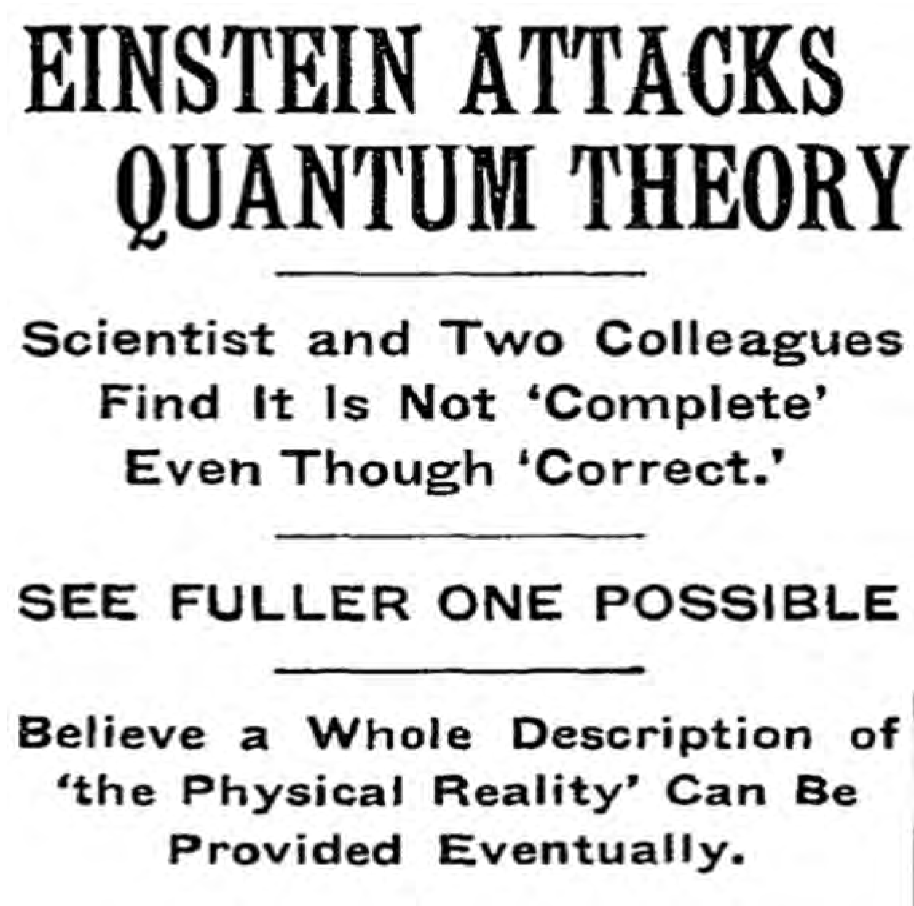
Einstein-Podolsky-Rosen (EPR) Paradox

- In 1934 Einstein-Podolsky-Rosen (EPR) wrote a paper where they described a variant of the previous experiment, and considered the outcome to be a "flaw" of the quantum theory.

- They spoke about a "spooky action at a distance", because the result of the remote experiment is known immediately after the first experiment.

- They also wrote a 1935 paper where they further argued on the "unphysical nature" of QM.

- This made it to the New York Times before publication.



EINSTEIN ATTACKS QUANTUM THEORY

Scientist and Two Colleagues Find It Is Not 'Complete' Even Though 'Correct.'

SEE FULLER ONE POSSIBLE

Believe a Whole Description of 'the Physical Reality' Can Be Provided Eventually.

- Einstein complained in writing to the NYT as they obtained "unauthorised information" (from Physical Review)

- A version of the argument using spins was later made by David Böhm in 1951.

• In 1964, John Bell, motivated by the EPR Paradox, and wanting to differentiate local hidden variable theories, derived his famous Bell inequalities, which is a sharp formulation of the EPR problem.

• In 1982, Alain Aspect did the first set of experiments using entangled photons to show that Bell's inequalities were violated, in agreement with QM.

• Many experiments followed and the first (almost) loophole-free experiment was done in 2015 by R. Hanson

• There has been a recent Bell-violation experiment using the light of two distant (8 Gly) quasars

D. Rauch et al.

Quantum Information

• The entanglement of two or more systems is a property of QM.

• It is a matter of "initial conditions" or dynamics.

• The reason that Quantum Computation has an important advantage over classical computation is due to entanglement.

• This has lead to the development of Quantum Information Theory: a transcript of Classical Information Theory to quantum systems.

• Key parts of Quantum Information Theory are

♠Quantum Cryptography

♠Quantum Computation

♠Quantum Teleportation

♠Quantum Complexity Theory

Entanglement Entropy

- The simplest measure of entanglement of two quantum sub systems A and B is the entanglement entropy.

- The general pure state of the total system $A \times B$ can be written as

$$|\psi\rangle = \sum_{a,m} C_{a,m} |a\rangle_A \otimes |m\rangle_B$$

- In general this is an entangled state of A and B.

- In the special case $C_{a,m} = D_a E_m$ then the state is not entangled:

$$|\psi\rangle = \left(\sum_a D_a |a\rangle_A\right) \otimes \left(\sum_m E_m |m\rangle_B\right)$$

- We imagine that we can not perform experiments on system B but only on A.

- To calculate what we expect for the experiments of system A, we must sum over all the possibilities in B we cannot measure.

• Because of this loss of information, the system A alone, as a quantum system is now described by a density matrix ρ_A.

$$(\rho_A)_{ba} = \sum_m C^*_{am}\, C_{bm} = (C^\dagger \cdot C)_{ba} \quad , \quad \rho_B = C \cdot C^\dagger$$

• Reminder: a diagonal density matrix

$$\rho_A = \begin{pmatrix} p_1 & 0 & 0 & \cdots \\ 0 & p_2 & 0 & \cdots \\ 0 & 0 & p_3 & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix} \quad , \quad \begin{matrix} p_1 & \leftrightarrow & |1\rangle_A \\ p_2 & \leftrightarrow & |2\rangle_A \\ p_3 & \leftrightarrow & |3\rangle_A \\ & \vdots & \end{matrix}$$

says that the system is in state $|\psi_1\rangle$ with probability p_1 and so on.

• These are CLASSICAL Probabilities.

• Always

$$Tr[\rho_A] \;=\; \sum_i p_i \;=\; 1$$

, but

$$Tr[\rho_A^2] \;=\; \sum_i p_i^2 \;\leq\; 1.$$

• $Tr[\rho_A^2] = 1$ if and only if the system is in a pure state ($p_1 = 1$, $p_2 = p_3 = \cdots = 0$).

• The entanglement density matrix ρ_A contains all the information of the entanglement of the system A with the system B.

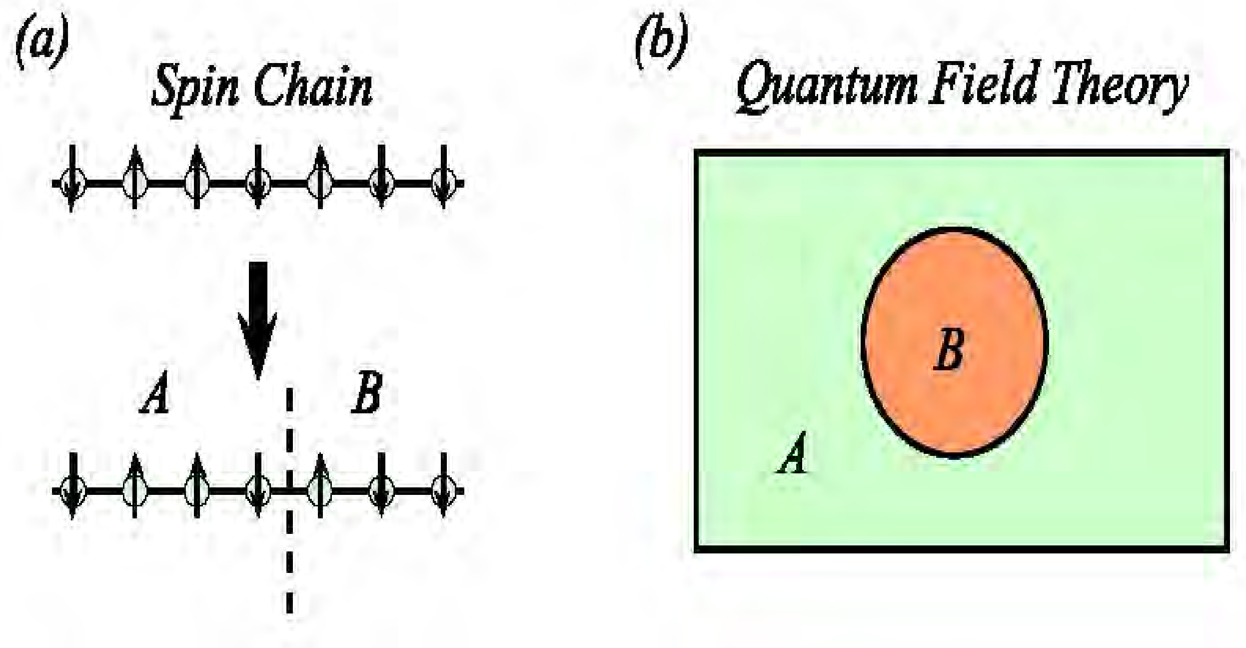
• The entanglement entropy is constructed as the standard Von Neumann entropy:

$$S_{entangl,A} \equiv -Tr[\rho_A \log(\rho_A)] = S_{entangl,B} = -Tr[\rho_B \log(\rho_B)] = -\sum_i p_i \log(p_i)$$

• It depends on the original state of the entangled system, as well as the subsystems themselves.

• When the original system $A \times B$ is in a non-entangled state, then $p_1 = 1$, $p_2 = p_3 = \cdots = 0$ and the entanglement entropy vanishes.

• If we consider the two systems to be spatial parts of the total system we have the following picture



• If the total system is in a pure state, $S_A = S_B$.

• Therefore the entanglement entropy cannot be extensive=not proportional to volume!

Thermodynamic Entropy

- Entropy was first defined in thermodynamics.

- A thermal system can be thought of as a complex system in a mixed quantum mechanical state, with thermal density matric

$$\rho_{thermal} = \frac{1}{Z} \sum_n e^{-\frac{E_n}{T}} \, |n\rangle\langle n|$$

- This is equivalent to the statement that the system has probability $p_n = e^{-\frac{E_n}{T}}$ to be in the energy eigenstate $|n\rangle$.

- It can also be thought of as a system in contact (=strong interaction) with a heat bath. The density matrix is due to the loss of information due to the interaction with a heat bath.

- It can also be a quantum mechanical system in a pure state, but at high energy.

• Lloyd's theorem indicates that for generic states in such a system, a microcanonical thermal density matrix is a very good approximation to the state of the system if the density of states is large.

Lloyd, 1987

• The thermal entropy is again given by a Von Neumann formula

$$S_{thermal} = -Tr[\rho_{thermal} \log(\rho_{thermal})] = \frac{1}{T} \frac{\sum_n \; E_n e^{-\frac{E_n}{T}}}{\sum_n \; e^{-\frac{E_n}{T}}}$$

• It is well known that the thermal entropy is extensive!

• There seems to be absolutely no relation between entanglement entropy and thermal entropy.

Properties of the Entanglement Entropy

• Entanglement properties and entropy of few body systems are easy to compute an analyse.

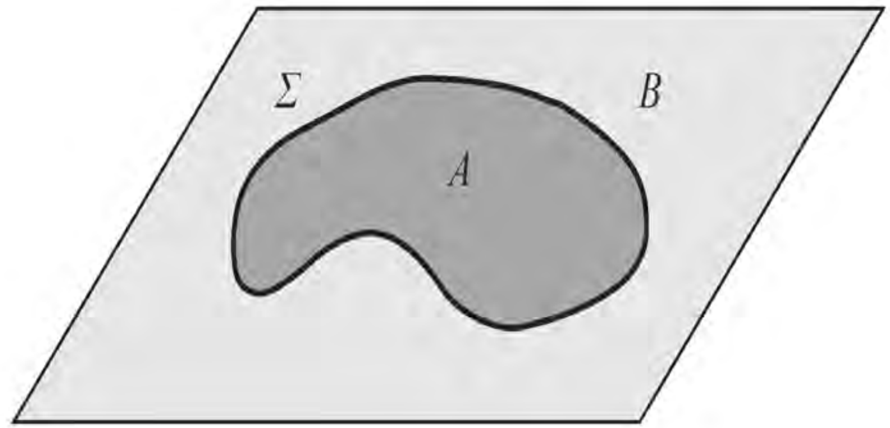
• However, the entanglement entropy in many body systems and Quantum Field Theory are notoriously difficult to compute.

• Even in free theories in $d > 2$ it is a VERY difficult observable to compute.

• Moreover, in continuum QFTs it is infinite.

• There are some general properties that can be proven without solving the theory:

• Consider a system and two spatial subsystems A, B.



• Then, the following inequality holds:

$$S_{A\cup B}+S_{A\cap B}\leq S_A+S_B.$$

• It is known as the strong subadditivity condition and its proof is very lengthy in QFT.

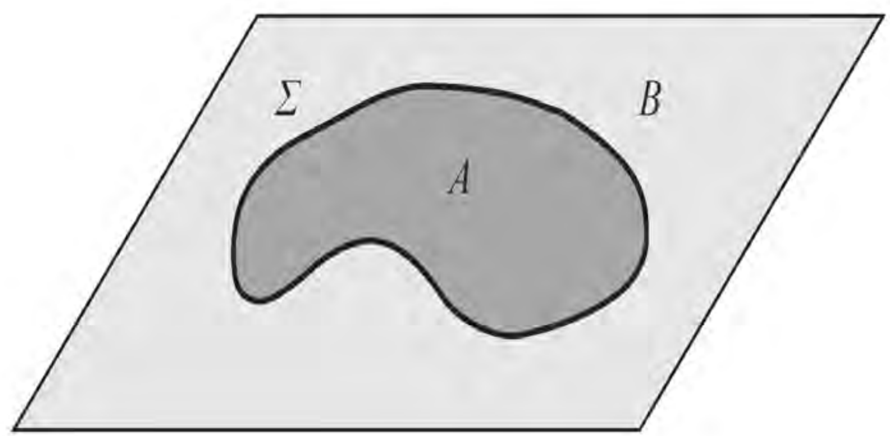
• There is no "understanding" why such inequalities hold!

Entanglement Entropy in continuum QFT

• In continuum QFT, we have a continuum number of degrees of freedom (a few per space point).

• We consider local QFTs because this is what we observe in nature. This means a space-time point interacts only with its "closest neighbors".

• When we separate space in A and its complement as in



we expect that most of the entanglement entropy of A with the complement (B) will come from the surface Σ separating A and B.

• Consider a QFT in 3+1 dimensions. We fix a time t_0, and we separate the three-dimensional space $\mathbb{R}^3$, into the interior of a sphere of radius R (this is region A) and its complement: the exterior of the sphere up to infinity.

• To calculate the entanglement entropy of the interior to the exterior degrees of freedom we must introduce a minimum length ϵ, (that could be a lattice spacing), (otherwise S_A is infinite):

$$S_A = a_2 \left(\frac{R}{\epsilon}\right)^2 + a_0 \log \frac{R}{\epsilon} + a_0' + \mathcal{O}(\epsilon)$$

• The leading (divergent) contribution scales as the area of the entangling surface (here a two-dimensional sphere).

• Although this is expected from the locality of the QFT, it rimes with another similar observation made by Bekenstein in 1972:

♠The thermal entropy of black holes is proportional to the area of the horizon rather than the volume that anyone would expect.

• F. Wilczek in the 90s, suggested there may be a connection between the entanglement entropy in QFT and the thermal entropy of Black Holes, but nobody understood why and how.

The Einstein-Rosen (ER) bridge

• After the EPR paradox papers in 1934-1935, Einstein and Rosen (ER) wrote another paper in 1935 on the Schwarszchild solution, that is describing the simplest non-rotating black hole:

$$ds^2 = -\left(1 - \frac{2GM}{r}\right) dt^2 + \frac{dr^2}{\left(1 - \frac{2GM}{r}\right)} + r^2 d\Omega_2^2$$

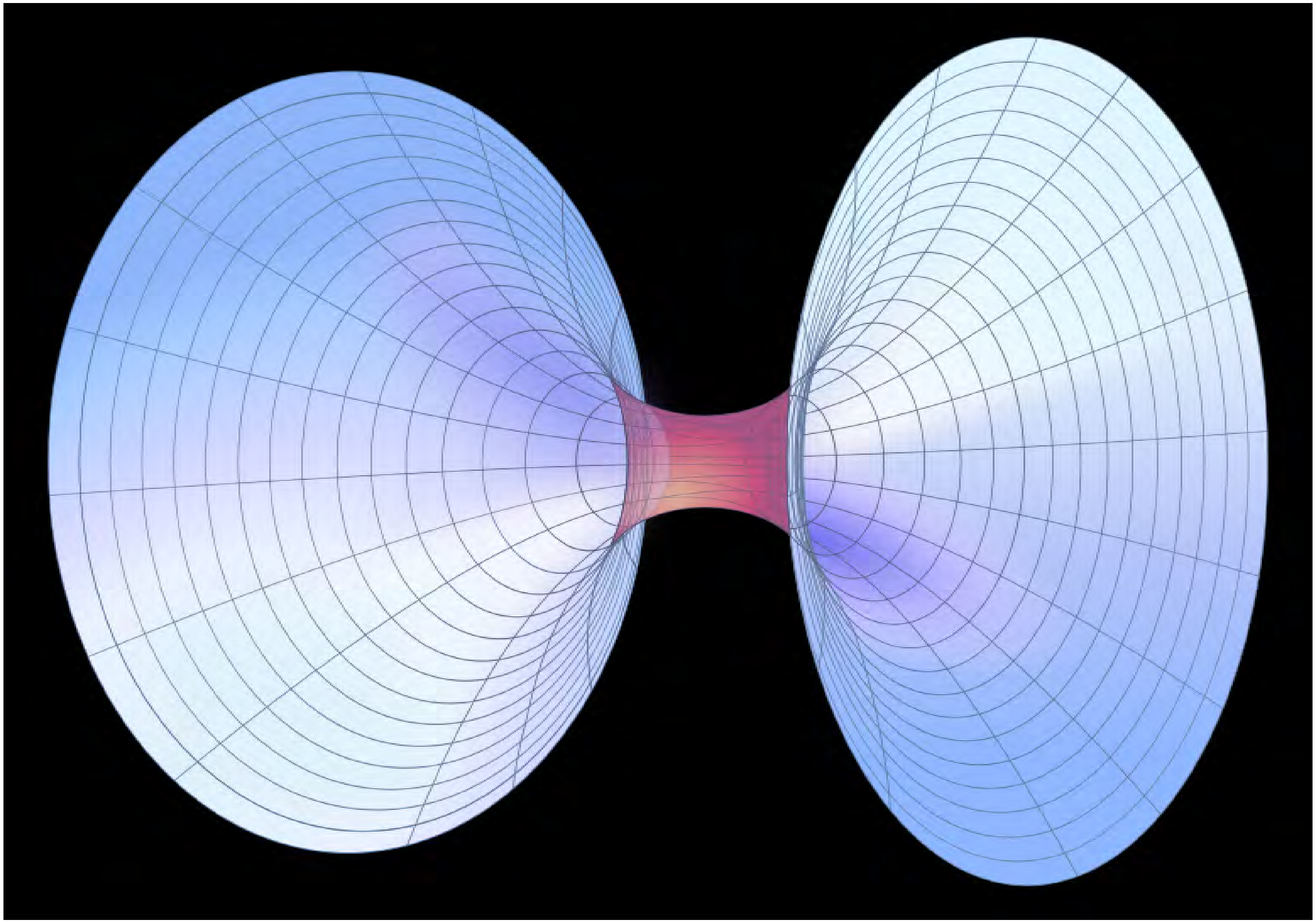
• In this coordinate system the horizon is at $r = 2GM$ and the singularity at $r = 0$.

• They considered the fixed $t = 0$ surface, and found that the metric there is regular, and it describes the geometry of a worm-hole (a name coined by J. Wheeler).

• They wanted to find ways of avoiding the Schwarzschild curvature singularity.

• This geometry became known as the Einstein-Rosen (ER) bridge (that linked two different universes).

• One side of this worm-hole is the usual asymptotic infinity, far from the source. The other is in another universe which is not visible in the Schwarzschild coordinates.



The Schwarzschild metric in Szekeres-Kruskal coordinates

• We charge coordinates from r, t to X, T so that

$$ds^2 = \frac{32(GM)^3}{r} e^{-\frac{r}{2GM}} (-dT^2 + dX^2) + r^2 d\Omega_2^2 \quad , \quad r \to r(T, X)$$

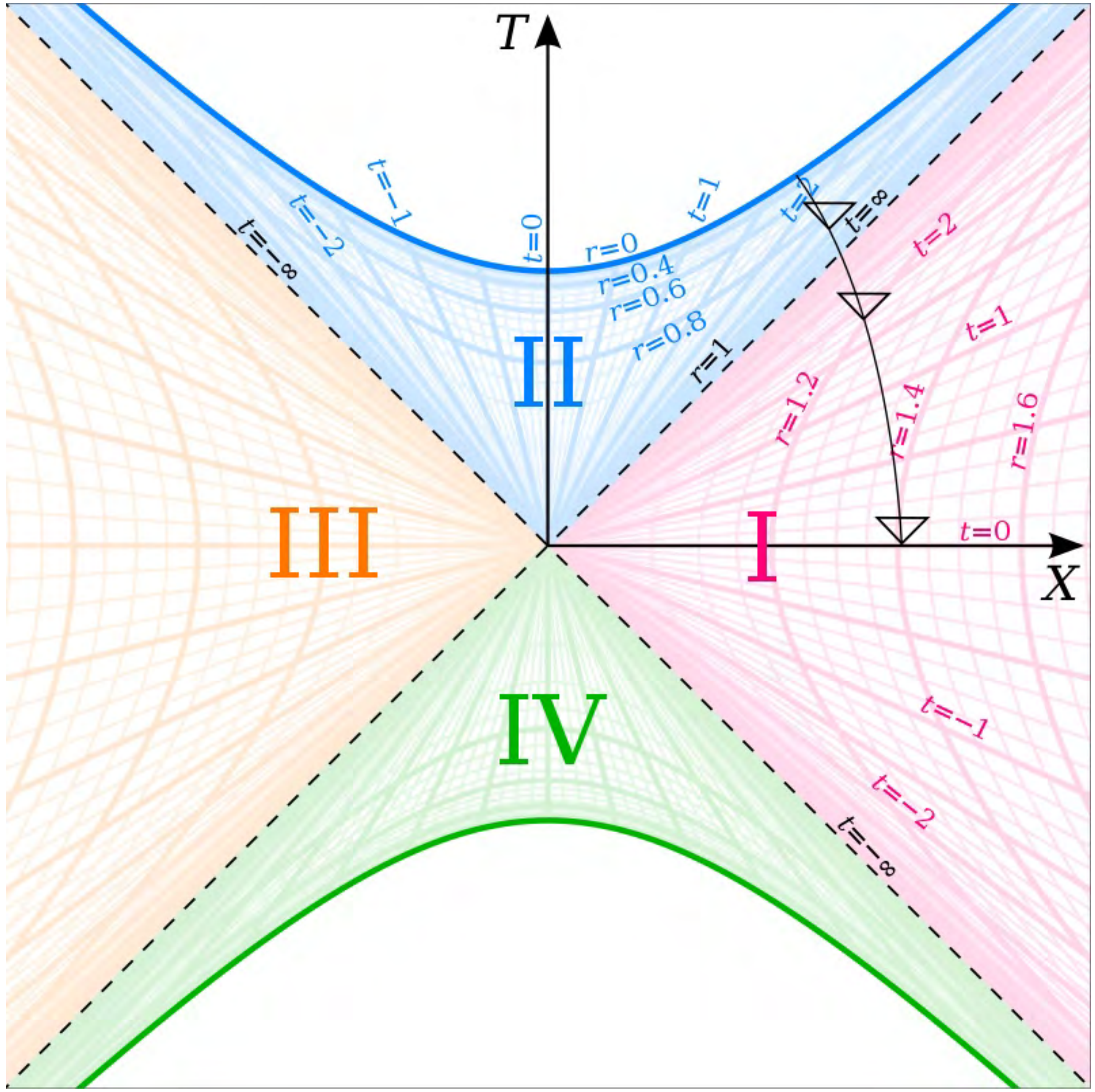
$$-\infty < T^2 - X^2 < 1 \quad , \quad -\infty < X < +\infty$$

• The horizon is at $X^2 = T^2$ and the singularity is at $T^2 - X^2 = 1$.

• This is a solution of the vacuum Einstein equations that maximally extends the Schwarzschild solution.

• It contains two disconnected boundaries isomorphic to the standard two-sphere at infinity.

• The $T = 0$ slice metric is the ER bridge (worm-hole).

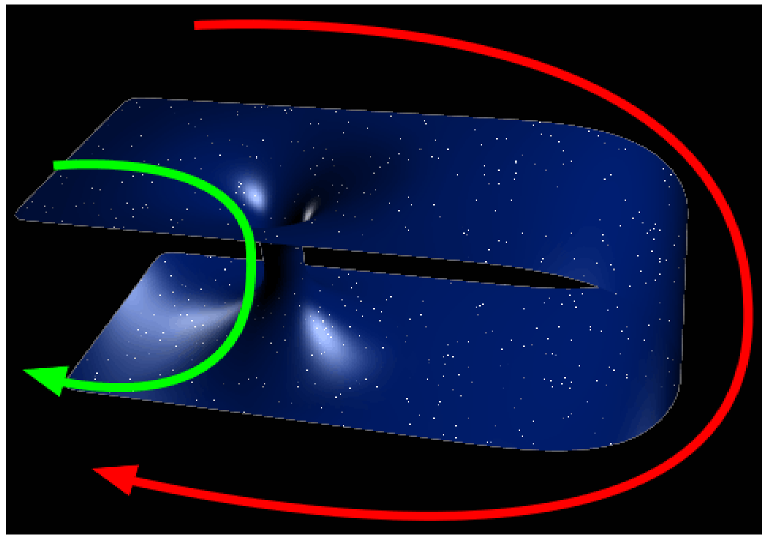


- The $T = 0$ three-dimensional space is the ER bridge (wormhole).

A quick recap on worm-holes

• Worm-holes are the darlings of science-fiction novels and movies.

• They are "tunnels" in space-time that connect different, (and sometimes distant) points in space-time.



• They ARE NOT traversable in classical Einstein gravity if matter behaves properly (null energy condition).

• But this changes in quantum gravity.

Jafferis+Zlokapa+Lykken+Kolchmeyer+Davis+Lauk+Neven+Spiropulu

The gauge-theory/gravity duality conjecture

• The gauge-theory/gravity duality is a duality that relates a string theory with a gauge theory.

• It also similar to say that it relates a quantum gravity theory (coming from the string theory) and a QFT (the gauge theory).

• The microscopic quantum degrees of freedom are the QFT degrees of freedom. There are many quantum degrees of freedom in number: $N \to \infty$.

• In the regime where the microscopic quantum degrees of freedom interact strongly, the dynamics of system is described by semiclassical gravity (and other interactions).

• One can think of this as follows: quantum (semiclassical) gravity emerges from the collective interaction of microscopic, strongly-interacting quantum degrees of freedom.

• In a sense, gravity is the analogue of what hydrodynamics is to a quantum many-body system.

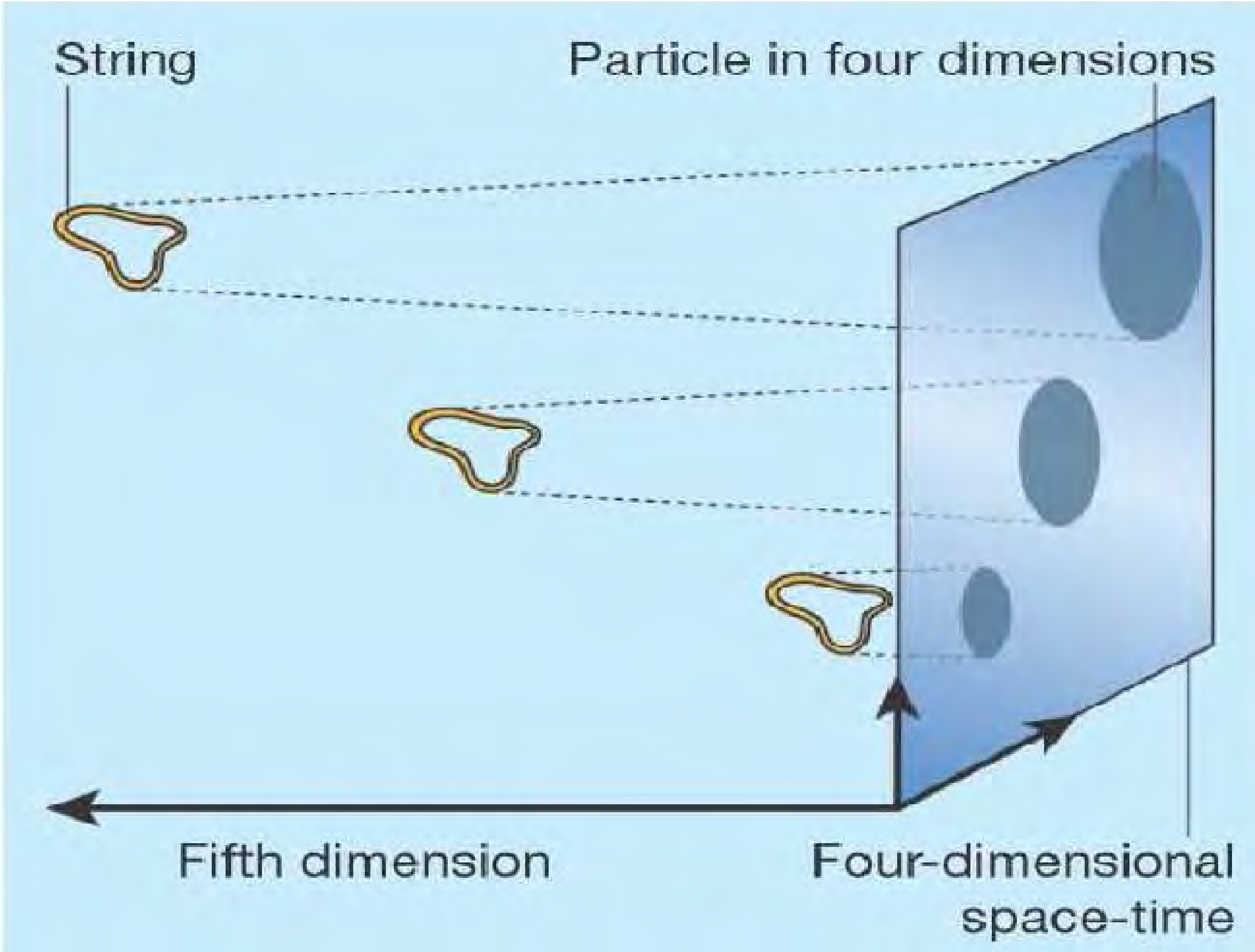
• The prime example is the AdS/CFT correspondence

Maldacena 1997

• It states that N=4 four-dimensional SU(N) gauge theory (gauge fields, 4 fermions, 6 scalars) is equivalent to ten-dimensional IIB string theory on $\text{AdS}_5 \times S^5$

• The gravitational theory has 6 extra emergent dimensions.

• The space (AdS_5) is non-compact and has a single boundary, at $r = 0$.



• This is a bulk (gravity) /boundary (QFT) correspondence or duality.

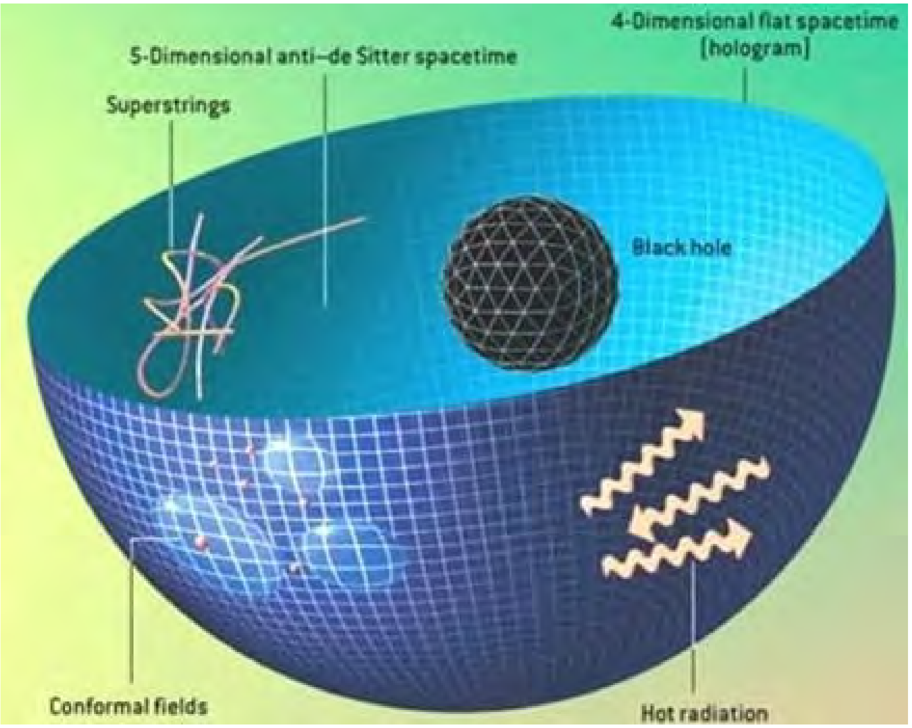
• The bulk geometry (AdS_5) corresponds to a state (the ground state) of the dual QFT.

The gauge-theory at finite temperature

• The finite temperature ground state of the gauge theory corresponds to a different solution in the dual string theory: the AdS-Black-hole solution *E. Witten, 1998*

• This solution has a horizon that is hiding the black-hole singularity

• The (thermal) Bekenstein entropy of the black holes is proportional to the horizon area. It is equal to the thermal entropy of the gauge theory.



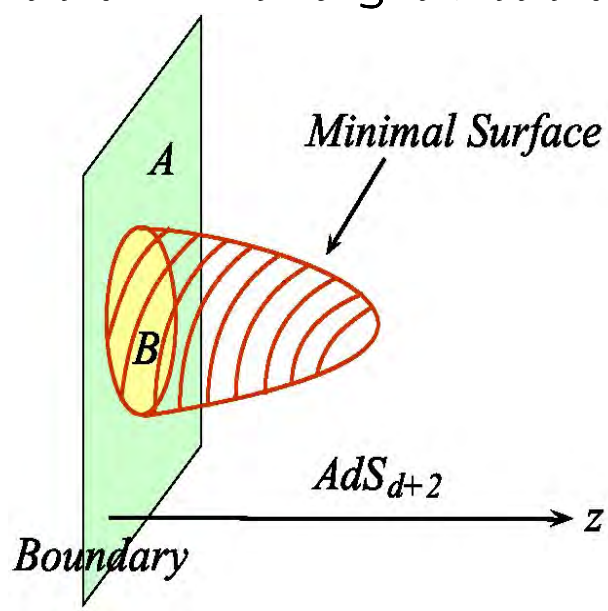
Entanglement Entropy (Ryu-Takayanagi formula)

- We consider a holographic QFT on $\mathbb{R}^4$.

- Separate a region B inside space $\sim \mathbb{R}^3$ and we would like to calculate the entanglement entropy associated to it.

- We have described this calculation earlier.

- What is the same calculation in the gravitational dual?



$$S_B = \frac{A_{minimal}}{4\ell_P^2}$$

Ryu-Takayanagi

• This formula was originally guessed, and then it was generalized to time-dependent contexts.

Hubeny-Rangamani-Takayanagi

• It was proven later by Lewkowitz and Maldacena to give the same answer as the QFT formula.

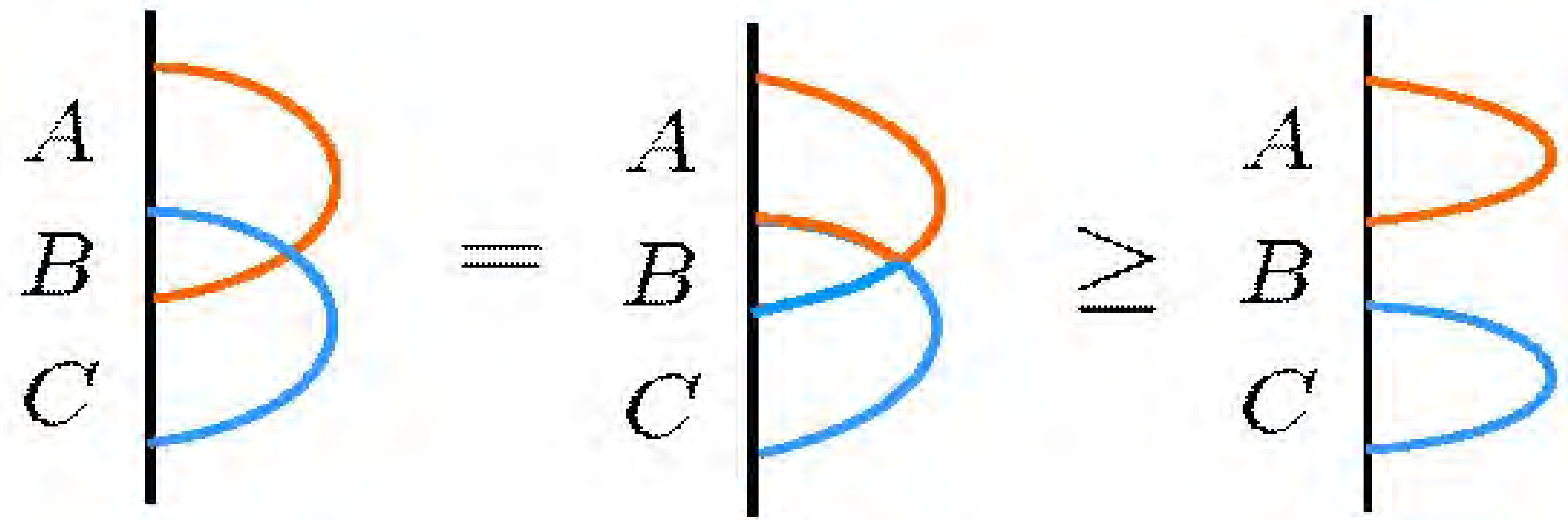
• It is amazingly simpler to compute it in gravity than in QFT.

• The UV divergence in the QFT is appearing as an IR divergence in gravity (near the boundary).

• The Strong Subadditivity property

$$S_{A\cup B}+S_{B\cup C}\geq S_A+S_C\,.$$

can be proven with one figure:



• This calculation can be generalized to a QFT at finite temperature.

• There are phase transitions in entanglement entropy that can be correlated with quantum phase transitions in the QFT.

Are entanglement and thermal entropies different?

• We have argued earlier that entanglement entropies and thermal entropies measure different things.

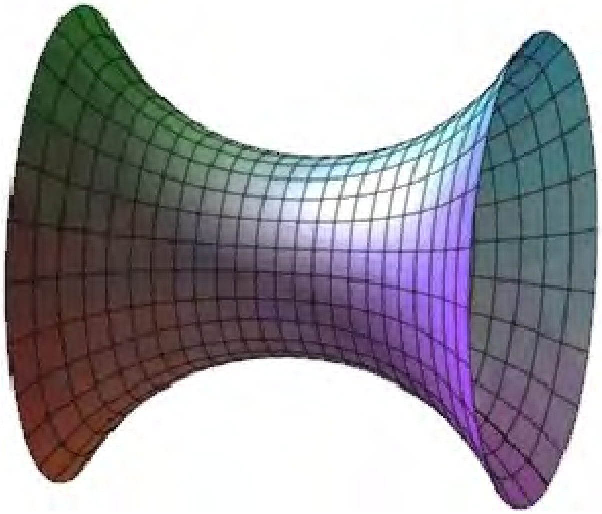
• Consider a QFT_4 on de Sitter space.

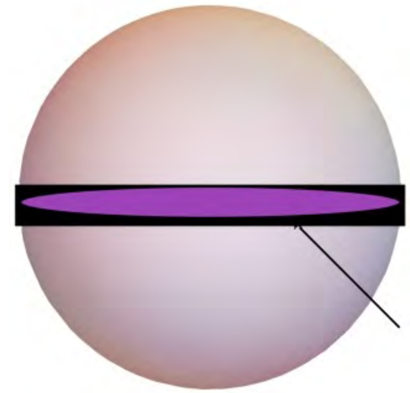
• This is the (very symmetric space) that solves Einstein's equations with a positive cosmological constant.

• Our universe today is closer and closer to de Sitter space.

• In global coordinates

$$ds^2 = -dt^2 + \cosh^2(Ht) d\Omega_3^2$$





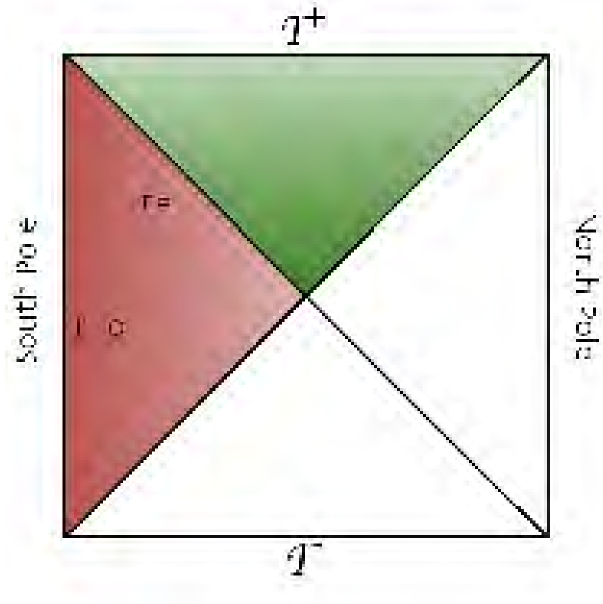
• We can compute the entaglement entropy S_{EE} of one of the two hemispheres of the spatial S^3 using the Ryu-Takayanagi formula.

• Consider another coordinate system (static coordinates)

$$ds^2 = -\left(1 - r^2 H^2\right) d\tau^2 + \left(1 - r^2 H^2\right)^{-1} dr^2 + r^2 d\Omega_2^2$$

• It describes a static observer in de Sitter.

• Now, there is a cosmological horizon at $r = \frac{1}{H}$.



• There is a Bekenstein thermal entropy associated with this horizon, $S_{thermal}$

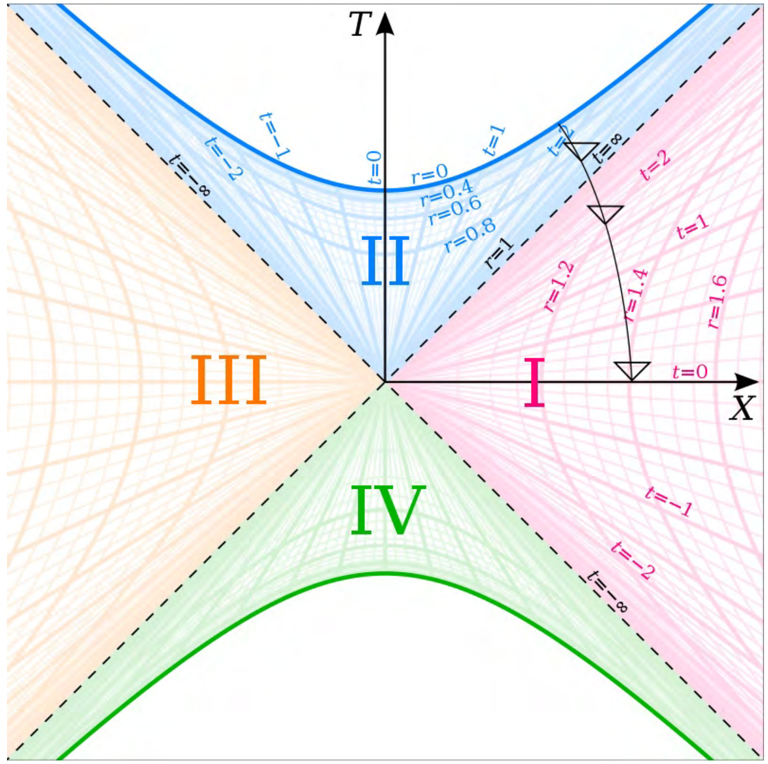
$$S_{EE} \quad = \quad S_{thermal}$$

Ghosh+E.K.+Nitti+Witkowski

• This realizes the suspicion of the 90's

• An entanglement entropy for one observer becomes a thermal entropy for another observer.

ER=EPR



• Now the two boundaries each correspond to (the same) QFT: $\to \quad QFT_L \times QFT_R$.

• The black-hole solution corresponds to the thermofield double state

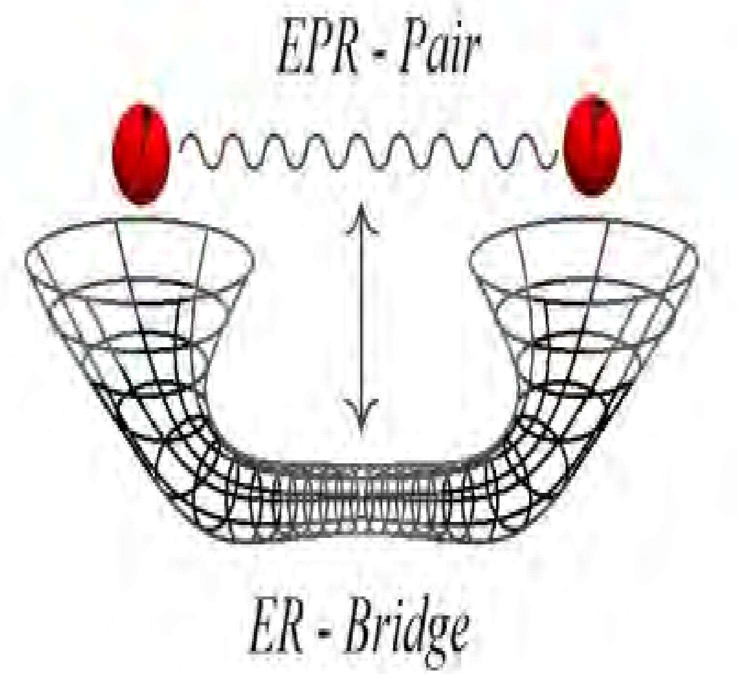
$$|\psi_{L,R}\rangle = \sum_n e^{-\frac{E_n}{2T}} \; |n\rangle_L \; \otimes \; |n\rangle_R$$

• This is a pure quantum state for the product of the two QFT's.

• In this state the two theories are strongly entangled.

• We have a direct correlation between strongly entangled pair of QFTs (EPR) and a ER bridge that connects the two parts of the entangled pair.

Maldacena+Susskind



“EPR=ER”

• Strong quantum entanglement between two systems, appears as a ER bridge (worm-hole) that connects them in emergent gravity.

• If one cuts in different ways the ER worm-hole, the smallest area gives the entanglement entropy.

• No observer in one theory can communicate, or make experiments in the other theory.

• We will therefore calculate the entanglement density matrix by summing over the theory on the left.

$$\rho_R = \sum_n e^{-\frac{E_n}{T}} \; |n\rangle_R \; {}_R\langle n|$$

• ρ_R is the thermal density matrix at temperature T, the Hawking temperature of the black hole.

• The entanglement entropy between the two black holes is equal to the thermal entropy of a single black hole.

• The extendend Kruskal solution with two asymptotic boundaries is the "purification" of the mixed state of a Schwarzschild black hole.

Quantum entanglement as space-time fabric

• It can be shown in full generality that entanglement entropy always satisfies a differential relation akin to the first law of thermodynamics.

Faulkner+Guica+Hartman+Myers+Van Raamsdonk

• With some mild assumptions this first law leads to the linearized Einstein equations for gravity.

Jacobson, Faulkner+Guica+Hartman+Myers+Van Raamsdonk

• It also provides a holographic map between the QFT and gravity data.

• The modular Hamiltonian associated to an entanglement entropy is generating a new "time" that is the time of a freely-falling observer in a gravitational field.

Jafferis+Lambrou

• All of this suggests that quantum entanglement of the underlying QFT degrees of freedom translates into the geometry of space-time and the laws of gravitation.

Black Holes again

• Black holes have the highest entropy density from any physical system.
Bekenstein

• They are the fastest scramblers of all systems.
Susskind

• They are the most chaotic quantum systems in nature as they saturate the quantum Lyapunov exponent bound.
Maldacena+Shenker+Stanford

• They are the most strongly entangled systems in nature.
Maldacena+Susskind

• They are potentially the most powerful quantum computers because of storage capacity and degree of entanglement.
Lloyd, Dvali

• And for (near) extremal black holes, quantum gravitational effects matter.
Kitaev, Maldacena,....

Epilogue

• In emergent quantum gravity, the space-time dimension and metric are avatars of the strong entanglement of the underlying (many) microscopic quantum degrees of freedom.

• The dynamics of the geometry emerges as thermodynamics and hydrodynamics emerge from quantum many-body systems.

• These are radical departures from Einstein's ideas that permeated our understanding of gravity and space-time till today.

• What do they tell us about the black-hole information paradox?

• What do they tell us about the second half-life of evaporating black-holes?

• Is the argument of Preskill+Harlow that the holographic correspondence is a quantum error correcting code generally applicable?

• Is there a precise correspondence between tensor networks and gravity?

• Is the Kitaev/Maldacena conjecture (any quantum system with a maximal Lyapunov exponent is dual to a black hole) true in general?

• How can we test these ideas in observable gravity?

• In all of the above, there were crucial contributions from other fields: Quantum Information (Kitaev, Preskill), Condensed Matter and Statistical Physics (Kitaev, Sachdev, Swingle) and recently the first experiments.

Jafferis+Zlokapa+Lykken+Kolchmeyer+Davis+Lauk+Neven+Spiropulu

• But, most of the work lies ahead.

THANK YOU!

A quantum traversable Wormhole=Quantum Teleportation

Jafferis+Zlokapa+Lykken+Kolchmeyer+Davis+ Lauk+Neven+Spiropulu

• It is known that the Sadchev-Ye-Kitaev model of 1d (quench) disordered Majorana fermions is dual to 2d quantum gravity on AdS_2.

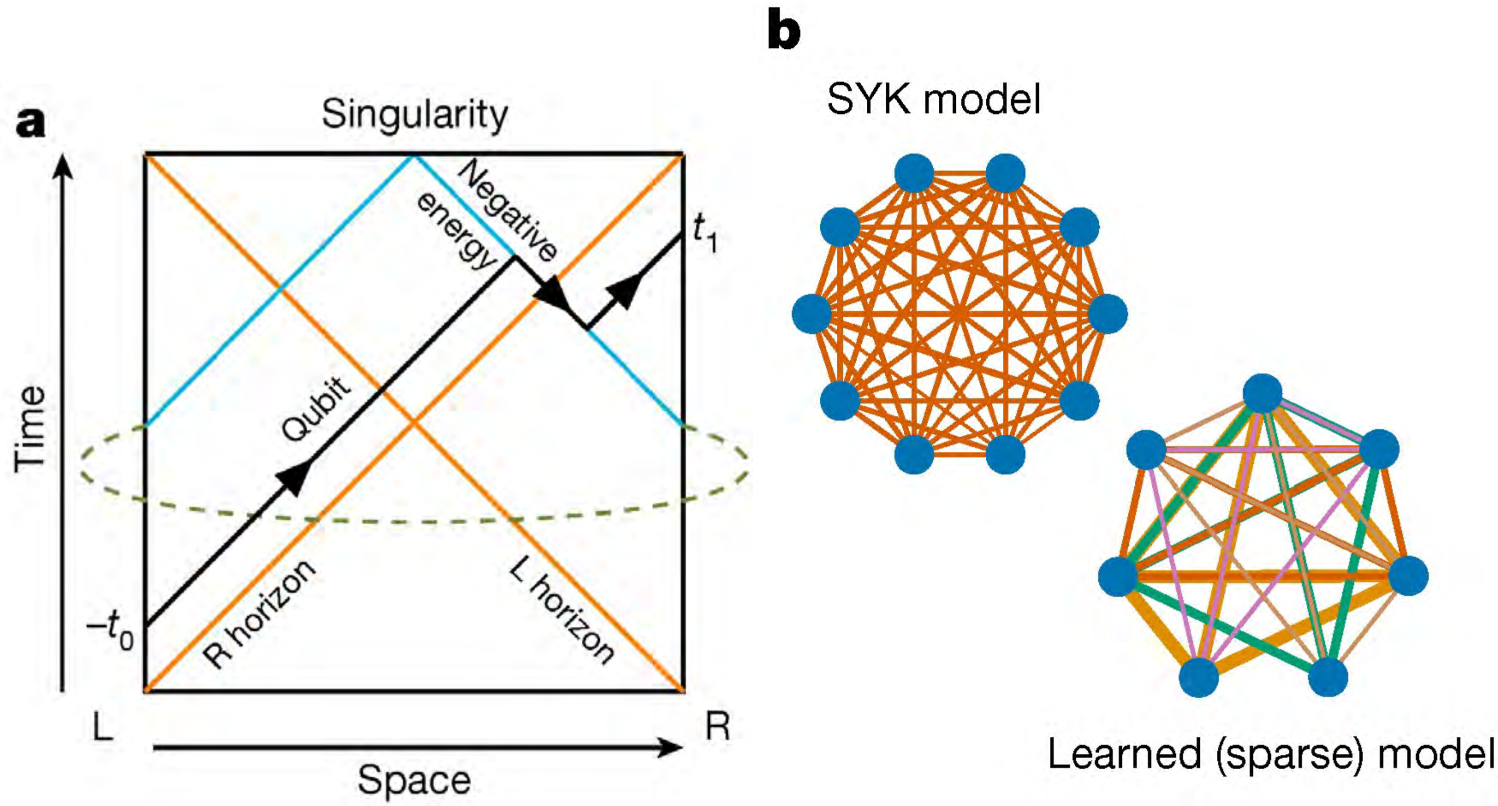
Kitaev, Maldacena+Shang

• Kitaev slightly modified the SY model, and showed by explicitly solving it that its quantum Liapunov exponent satisfied the Maldacena bound, suggesting that it is dual to a black hole in two dimensions. This is known as the Jackiw-Teitelboim or AdS_2 black hole.

• One can consider two decoupled SYK models at finite temperature to construct an entangled pair of two AdS_2 black holes.

- So far no traversable wormhole exists between the two SYK models.

- Jafferis, Gao and Wall had shown that a weak coupling between the two SYK Models can induce quantum-induced traversability of the semiclassical wormhole which is classically not traversable because of the singularity.



- This setup of a quantum-traversable wormhole was realized approximately and experimentally

• Due to experimental limitations the SYK models were "sparsified".

• Despite this, it was shown that the simplified model has all the tell-tale properties of SYK model namely:

♠perfect size winding,
♠coupling on either side of the wormhole that is consistent with a negative energy shockwave,
♠a Shapiro time delay,
♠causal time-order of signals emerging from the wormhole,
♠and scrambling and thermalization dynamics.

• The experiment was run on the 53-qbit Google Sycamore quantum processor.

Tensor Networks, Entanglement and Anti-de Sitter Space

• Brian Swingle was between 2006-2010 a graduate student of famous condensed matter physicist Xiao Gang Wen at MIT.

• His project was the study of tensor networks as ansätze for solving strongly-coupled 1d problems.

• His curiosity brought him to sit in a string theory class, where he learned about the AdS/CFT correspondence.

• In May 2009 he posted a solo paper on the ArXiV, where he argued that Tensor Networks for quantum critical systems generate a discretization of AdS space via their entanglement structure.

• The paper was submitted to the leading journal of the field (JHEP) and was rejected after two years of back and forth.

• It was resubmitted in 2011 to Phys Rev D, and was accepted after one year.

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Entanglement renormalization and holography

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We show how recent progress in real space renormalization group methods can be used to define a generalized notion of holography inspired by holographic dualities in quantum gravity. The generalization is based upon organizing information in a quantum state in terms of scale and defining a higher-dimensional geometry from this structure. While states with a finite correlation length typically give simple geometries, the state at a quantum critical point gives a discrete version of anti-de Sitter space. Some finite temperature quantum states include black hole-like objects. The gross features of equal time correlation functions are also reproduced.

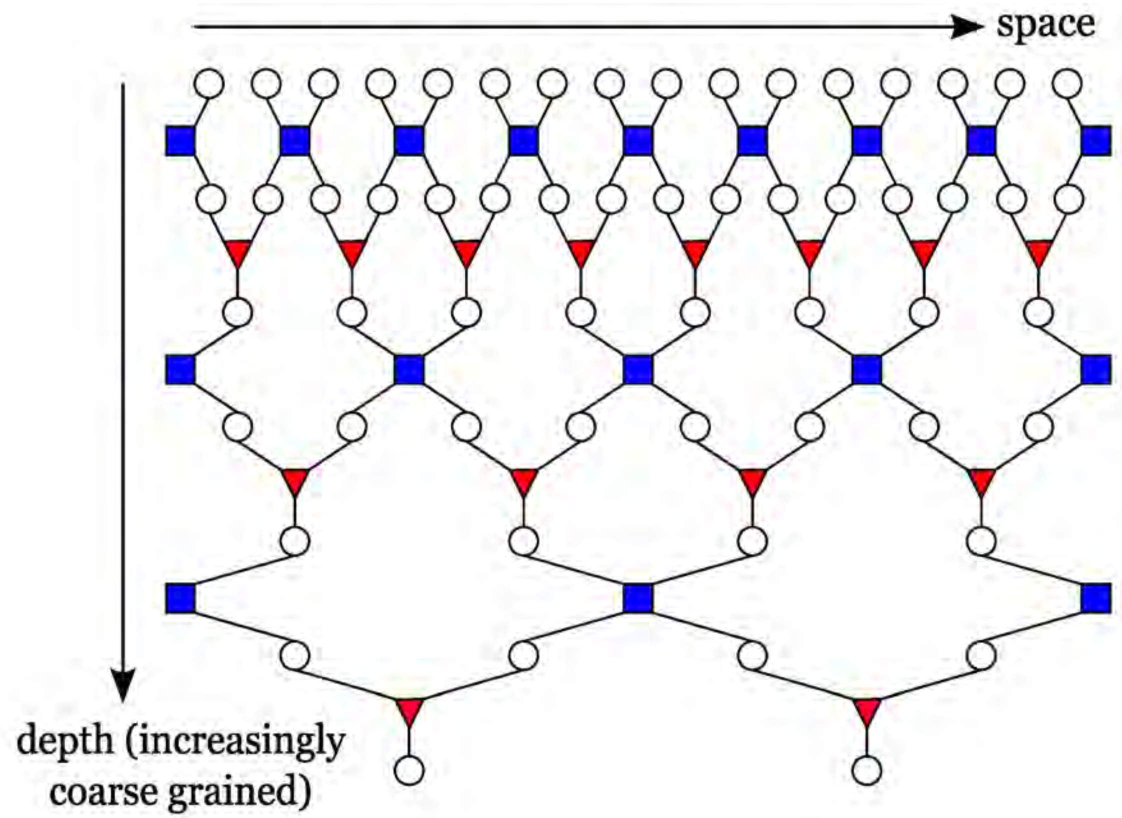

FIG. 1 (color online). The tensor network structure of entanglement renormalization. Circles are lattice sites at various coarse-grained scales. Squares with four lines are unitary disentanglers and triangles with three lines are isometric coarse graining transformations. The network shown here represents a $2 \to 1$ coarse graining scheme and has a characteristic fractal structure. In principle, each tensor can be different, but requiring translation and scale invariance provides strong constraints.

Quantum Chaos, Scrambling and Black Holes

• A classical chaotic system is one where infinitesimal changes in the initial conditions are exponentially magnified in the later evolution (the "butterfly effect").

$$\{x(t), p(0)\}_{PB} = \frac{\delta x(t)}{\delta x(0)} \sim e^{\lambda t}$$

• λ is known as the (classical) Lyapunov exponent and controls the classical chaotic behavior of a system.

• The quantum analogue of this condition defines the quantum Lyapunov exponent.

$$\{x(t), p(0)\}_{PB} \to [\widehat{x}(t), \widehat{p}(0)] \quad : \quad \langle\psi| \; [\widehat{x}(t), \widehat{p}(0)]^2 \; |\psi\rangle \sim e^{\lambda t}$$

and this can be generalized to arbitrary "simple" operators in a QFT:

$$-\langle \; [V(t), W(0)]^2 \; \rangle_\beta \sim \begin{cases} \frac{1}{N}, & t \leq t_{\text{relax}}, \\ \frac{1}{N} e^{\lambda t}, & t_{\text{relax}} \leq t \leq t_{\text{scrambling}}. \end{cases}$$

- This is called scrambling and is associated to quantum chaos.

- This is correlated with the behavior of the out-of-time ordered correlator (OTOC)

$$\langle\, V(t)W(0)V(t)W(0)\,\rangle_\beta \simeq 1-\frac{e^{\lambda t}}{N}$$

- The Maldacena-Shenker-Stanford bound on quantum chaos:

$$\lambda \;\; \leq \;\; \frac{2\pi T}{\hbar}$$

- Black holes saturate the bound!

- Kitaev has revamped the 1d-model of Sachdev-Ye with Majorana fermions and quenched disorder and has shown that it saturates the Maldacena chaos bound above.

- This 1-d theory in the thermal state, was shown to be dual to 2d JT gravity and in particular to the AdS_2 back hole.

- There is a conjecture: any system that saturates the Maldacena chaos bound is dual to an gravitational Black hole.

The modular Hamiltonian of entanglement

• Given a region A, we compute the entanglement density matrix ρ_A associated to the region A, after summing all the rest in the theory that does not belong to A.

• We define the modular Hamiltonian associated to the region A as H_A that satisfies

$$e^{-H_A} \equiv \rho_A \quad , \quad S_A = Tr[H_A \, e^{-H_A}]$$

• H_A is Hermitian but possibly unbounded and has UV divergences etc, but can be handled most of the time in more refined entanglement observables (like relative entropy).

• It can be used to evolve the operators O of the subsystem A in a fictitious time, α

$$O(\alpha) \equiv e^{i\alpha H_A} \, O \, e^{-i\alpha H_A} \quad , \quad Tr[\rho_A O(\alpha)] = Tr[\rho_A O]$$

• For every entangling region A we have a modular Hamiltonian!.

• It has been proposed, that the modular Hamiltonian of the entanglement entropy between the black hole and a freely falling observer generates the time coordinate for this observer.

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