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Operator Algebra of the N=1 Super Wess-Zumino Model[1]

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ABSTRACT

The projective Ward identities for the super Kac-Moody algebra are solved in order to determine the 2- and 3-point functions in the $N = 1$ superspace. Linear super-equations are derived for the correlation functions of degenerate operators. These equations are solved for the 3- and 4-point functions, proving the closure of the operator algebra of the super Wess-Zumino model.

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Recently, there has been a lot of interest in Kac-Moody algebras from both the mathematical and the physical point of view [1]. In particular, Kac-Moody algebras are seen to play an important role in conformally invariant two-dimensional models with continuous symmetries [2], [3], as well as in string theories [4],[5].

Typical models exhibiting such an algebra are the Wess-Zumino models on group manifolds, describing string propagation in the group manifold. Their supersymmetric version has been studied recently [6], and a new structure, that of a ($N = 1$) super Kac-Moody algebra, has emerged. This algebra is essential to describe superstring propagation in a group manifold [2].

In this letter, we derive the field transformation under a super Kac-Moody algebra, and we solve the projective Ward identities for the 2- and 3-point functions. We focus attention on the degenerate representations of the algebra that appear in the super Wess-Zumino model, and we derive the linear equations for the correlation functions of the degenerate fields. We solve these equations for the 3-point function, obtaining constraints on the dimensions of the fields that may exist in the theory. We thus show that the operator algebra of the degenerate fields closes in the same way as in the ordinary Wess-Zumino model. We also solve these equations to determine the 4-point function, which is the first non-trivial Green function. Some implications are also discussed.

The super Kac-Moody algebra is generated by the current superfield $\mathcal{J}^a \equiv \psi^a(z) + \theta J^a(z)$. In terms of the fourier modes of the supercurrent, this algebra is

$$[J^a_m, J^b_n] = if^{abc}J^c_{m+n} + k\delta^{ab}m\delta_{m+n,0} \quad , [J^a_m, \psi^b_r] = -if^{abc}\psi^c_{m+r} \quad , \left\{\psi^a_r, \psi^b_s\right\} = \delta^{ab}\delta_{r+s,0} \quad , \qquad (0.1)$$

where f^{abc} are the structure constants of a semi-simple Lie group G. We also have $(J^a_m)^\dagger = J^a_{-m}$ and $(\psi^a_r)^\dagger = \psi^a_{-r}$. One distinguishes between two sectors, the NS sector, where $\psi^a(z)$ is anti-periodic on the cylinder, and the R sector, where $\psi^a(z)$ is periodic on the cylinder. In this letter we shall only consider the NS sector. It is convenient to pass from the cylinder to the plane through the super-analytic transformation $(lnz, z^{-1/2}\theta) \rightarrow (\tau + i\sigma, \theta)$. Then in the NS sector fermionic fields are single-valued on the plane whereas in the R sector the fermionic fields are double-valued on the plane.

A theory invariant under the algebra (0.1) also has an $N = 1$ superconformal invariance. The generators of the superconformal algebra can be constructed from those of the super Kac-Moody algebra (in the Sugawara form),

$$L_n \equiv \frac{1}{2}\sum_{r\in \boldsymbol{Z}+\frac{1}{2}} (r+\frac{1}{2}) \ : \ \psi^a_{n-r}\psi^a_r : + \frac{1}{2k}\sum_{m\in\boldsymbol{Z}} \ : \ \tilde{J}^a_{n-m}\tilde{J}^a_m \ : \quad , \qquad (2a)$$

$$G_r \equiv -\frac{1}{\sqrt{k}}\sum_{m\in\boldsymbol{Z}} \ : \ \psi^a_{r-m}\tilde{J}^a_m \ : \ + \frac{1}{6\sqrt{k}}if^{abc}\sum_{m\in\boldsymbol{Z},r'\in\boldsymbol{Z}+\frac{1}{2}} \ : \ \psi^a_{r-m}\psi^b_{m-r'}\psi^c_{r'} \ : \quad , \qquad (2b)$$

[2]Despite some problems associated with this scenario (broken world-sheet supersymmetry in the Ramond sector of the group manifold), there is serious hope that some phenomenologically relevant string theory will be constructed.

$$\tilde{J}^a_m \equiv \tilde{J}^a_m - \frac{i}{2} f^{abc} \sum_{r\in\boldsymbol{Z}} + \frac{1}{2} \ : \ \psi^b_{m-r}\psi^c_r \ : \quad , \tag{2c}$$

where $\tilde{J}^a_m$ is the "bosonic" current. It can be easily seen that

$$\left[\tilde{J}^a_m \ , \quad \psi^b_r\right] = 0 \quad , \tag{3a}$$

$$\left[\tilde{J}^a_m \ , \quad \tilde{J}^b_n\right] = i f_{abc}\tilde{J}^c_{m+n} + (k - \frac{1}{2}c_A)\delta^{ab}_m\delta_{m+n,0} \quad , \tag{3b}$$

where $f^{acd}f^{bcd} = c_A\delta^{ab}$. The Fock in-vacuum $|0\rangle$ is defined as the state annihilated by the generators $\tilde{J}^a_n$ $(n \geq 0)$ and ψ^a_r $(r > 0)$. We end up with the semidirect product of the $N = 1$ superconformal algebra and the super Kac-Moody algebra defined by the commutation relations (0.1) together with:

$$\begin{aligned}
[L_m, L_n] &= (m-n)\ L_{m+n} + \frac{1}{8}\hat{c}(m^3 - m)\delta_{m+n,0} \quad , \\
[L_m, G_r] &= (\frac{1}{2} - r)G_{m+r} \quad , [L_m, J^a_n] = -nJ^a_{m+n} \quad , \\
[L_m, \psi^a_r] &= -(\frac{1}{2}m + r)\psi^a_{m+n} \quad , \\
\{G_r, G_s\} &= 2L_{r+s} + \frac{1}{2}\hat{c}(r^2 - \frac{1}{4})\delta_{r+s,0} \quad , \\
[G_r, J^a_m] &= \sqrt{k}m\psi^a_{m+r} \quad , \{G_r, \psi^a_s\} = -(\frac{1}{\sqrt{k}})J^a_{r+s} \quad ,
\end{aligned} \tag{0.4}$$

where $\hat{c} = (1 - \frac{c_A}{3k})D$, D being the dimension of the group G. We focus on the left sector of the theory, the full theory being the direct product of the left and right sectors. The highest weight vectors of this algebra (primary states) are labeled by the eigenvalues of the zero modes L_0, J^a_0,

$$L_0|R_i\rangle = \Delta|R_i\rangle \quad , \qquad J^a_0|R_i\rangle = (T^a_{ij})|R_j\rangle \quad , \tag{0.5}$$

where R denotes an irreducible representation of the group G. We also have $L_n|R\rangle = J^a_n|R\rangle = G_r|R\rangle = \psi^a_r|R\rangle = 0$, for $n, r > 0$. These states are generated by the action of superfield operators, called primary superfields, on the in-vacuum,

$$|R_i\rangle \equiv \Phi^R_i(0)|0\rangle \quad , \tag{6a}$$

where

$$\Phi^R_i(z,\theta) = \Phi^R_i(z) + \theta\psi^R_i(z) \quad . \tag{6b}$$

The algebra acts on the primary superfields as follows:

$$\begin{aligned}
\left[L_m, \Phi^R_i(z,\theta)\right] &= z^{m+1}\partial_z\Phi^R_i(z,\theta) + (m+1)z^m(\Delta + \frac{1}{2}\theta\partial/\partial\theta)\Phi^R_i(z,\theta) \quad , &(7a)\\
\left[G_r, \Phi^R_i(z,\theta)\right] &= z^{r+\frac{1}{2}}(\partial/\partial\theta - \theta\partial/\partial z)\Phi^R_i(z,\theta) - 2\Delta(r + \frac{1}{2})z^{r-\frac{1}{2}}\theta\Phi^R_i(z,\theta) \quad , &(7b)\\
\left[J^a_m, \Phi^R_i(z,\theta)\right] &= z^m(T^a_R)_{ij}\Phi^R_i(z,\theta) \quad , &(7c)\\
\left[\psi^a_r, \Phi^R_i(z,\theta)\right] &= \frac{1}{\sqrt{k}}z^{m-\frac{1}{2}}(T^a_R)_{ij}\theta\Phi_j(z,\theta) \quad . &(7d)
\end{aligned}$$

Here Δ is the conformal weight of the superfield Φ, defined in equation (5). The algebra above follows from the transformation of the superfield under the superconformal group and the Jacobi identities.

The theory is invariant in particular under the global superconformal group, $OSP(2|1)$, generated by $G_{\pm 1/2}$, $L_{\pm 1}$, L_0, due to the fact that the vacuum is also $OSP(2|1)$ invariant. We can derive appropriate Ward identities for the correlation functions reflecting the invariance mentioned above. The procedure is to insert a generator of $OSP(2|1)$ in a correlation function acting on the in-vacuum and move it to the left using the commutation relations (7). Let's consider the 2-point function. The Ward identities from global superconformal invariance fix its form to be [7],

$$\langle 0|\Phi_i^{R_1}(z_1,\theta_1)\Phi_j^{R_2}(z_2,\theta_2)|0\rangle = \frac{A_{ij}}{z_{12}^{\Delta_1+\Delta_2}}\delta_{\Delta_1,\Delta_2} \quad , \tag{0.8}$$

where $z_{12} = z_1 - z_2 - \theta_1\theta_2$. The vacuum is also invariant under global G-transformations, that is, the zero mode J_0^a annihilates the vacuum. The Ward identity for the zero mode of $J^a(z)$ implies

$$(T_{R_1}^a)_{ik}A_{kj} + (T_{R_2}^a)_{jk}A_{ik} = 0 \quad , \tag{0.9}$$

with a solution

$$A_{ij} \sim \langle R_1, R_2, i, j|1, 0\rangle \quad , \tag{0.10}$$

which is the Clebsch-Gordan coefficient of the projection of $R_1 \times R_2$ on the singlet.

The 3-point function is constrained by the superconformal invariance to have the form [7]

$$\langle 0|\Phi_i^{R_1}(z_1,\theta_1)\Phi_j^{R_2}(z_2,\theta_2)\Phi_k^{R_3}(z_3,\theta_3)|0\rangle = \left[\frac{A_{ijk}}{z_{12}^{\Delta_{12}}z_{13}^{\Delta_{13}}z_{23}^{\Delta_{23}}}\right](1+a\hat{\eta}) \quad , \tag{0.11}$$

where

$$\hat{\eta} = (z_{12}z_{13}z_{23})^{-1/2}(\theta_1 z_{23} - \theta_2 z_{13} + \theta_3 z_{12} + \theta_1\theta_2\theta_3) \tag{0.12}$$

is the only combination of the coordinates that is invariant under the global superconformal group $OSP(2|1)$, and squares to zero. Thus a is an extra undetermined Grassmann parameter.

The current Ward identity is in this case

$$(T_{R_1}^a)_{i\ell}A_{\ell jk} + (T_{R_2}^a)_{j\ell}A_{i\ell k} + (T_{R_3}^a)_{k\ell}A_{ij\ell} = 0 \quad , \tag{0.13}$$

with the solution

$$A_{ijk} \sim \langle R_1, R_2, i, j|R_3, k\rangle \quad , \tag{0.14}$$

the appropriate Clebsch-Gordan coefficient. The condition for the 3-point function to be non-zero is that the primary superfield Φ_3 to be contained in the operator product of Φ_1, and Φ_2. Then the z-independent part of the 3-point function is the operator product coefficient multiplying Φ_3 in the expansion of the product $\Phi_1 \times \Phi_2$. Let us remark that, unlike the non-supersymmetric case, there are two operator-product coefficients to be determined here, one corresponding to the overall normalization the other corresponding to the free parameter multiplying $\hat{\eta}$.

A representation of the semi-direct product of a super Kac-Moody and a superconformal algebra is said to be degenerate if there is a secondary state in it, (that is a state generated from the hwv by the action of lowering operators), that has the properties of a highest-weight vector, i.e., it is annihilated by all the raising operators of the algebra. The corresponding primary superfield is also said to be degenerate. This state is null. Its inner product with any other state in the representation generated by the initial primary superfield, vanishes identically. Thus the sub-representation generated by the null vector can be consistently set to zero. In particular, its correlation functions with all the other primary superfields vanish. The Sugawara form of the superconformal generators implies that such a state is of the form

$$|\chi\rangle = \left[\sqrt{k} G_{-1/2} + T_R^a \psi_{-1/2}^a\right] \ |R\rangle \quad . \tag{0.15}$$

It is easy to verify that $|\chi\rangle$ is annihilated by all the raising operators, provided that its dimension is, $\Delta = \frac{c_R}{2k}$, where $(T_R^a T_R^a)_{ij} = c_R \delta_{ij}$.

The existence of degenerate representations in a theory, is of prime importance because in such a case the correlation functions of a degenerate superfield satisfy additional linear (super)differential equations which allow one to determine them completely.

To make the above more precise consider the 3-point function with one of the fields, $\Phi_i^{R_3}$ say, being degenerate. Taking advantage of the invariance of the correlation functions under global superconformal transformations, we can perform a translation and a supersymmetry transformation, to bring it into the form

$$\begin{aligned} F_{ijk} &\equiv \langle 0|\Phi_i^{R_1}(\tilde{z}_1, \theta_1)\Phi_j^{R_2}(\tilde{z}_2, \tilde{\theta}_2)\Phi_k^{R_3}(0,0)|0\rangle \\ &= \left[A_{ijk}/(\tilde{z}_1 - \tilde{z}_2 - \tilde{\theta}_1\tilde{\theta}_2)^{\Delta_{12}} \ \tilde{z}_1^{\Delta_{13}} \ \tilde{z}_2^{\Delta_{23}}\right](1 + a\hat{\eta}) \quad , \end{aligned} \tag{0.16}$$

where

$$\begin{aligned} \tilde{z}_1 &= z_1 - z_3 - \theta_1\theta_3 \quad , \quad \tilde{\theta}_1 = \theta_1 - \theta_3 \quad , \\ \tilde{z}_2 &= z_2 - z_3 - \theta_2\theta_3 \quad , \quad \tilde{\theta}_2 = \theta_2 - \theta_3 \quad . \end{aligned} \tag{0.17}$$

Using the fact that the field Φ^o corresponding to the null state $|\chi\rangle$ has vanishing correlation functions with all other fields, we obtain

$$\langle 0|\Phi_i^{R_1}(\tilde{z}_1, \tilde{\theta}_1)\Phi_j^{R_2}(\tilde{z}_2, \tilde{\theta}_2)(\sqrt{k}G_{-1/2}\delta_{k\ell} + (T_{R_3}^a)_{k\ell}\psi_{-1/2}^a)\Phi_\ell^{R_3}(0,0)|0\rangle = 0 \quad . \tag{0.18}$$

Commuting the generator of the algebra through to the left using Eq. (7), we arrive at the following super-equation for the 3-point function (we drop the tildes from now on):

$$k\left[\sum_{i=1}^{2}\left(\frac{\partial}{\partial\theta_i} - \theta_i\frac{\partial}{\partial z_i}\right)\right] F_{ijk} + \frac{\theta_1}{z_1}\left(T_{R_3}^a\right)_{km}\left(T_{R_1}^a\right)_{i\ell} F_{\ell jm} + \frac{\theta_2}{z_2}\left(T_{R_3}^a\right)_{km}\left(T_{R_2}^a\right)_{j\ell} F_{i\ell m} = 0 \quad . \tag{0.19}$$

Eq.(0.19) implies that the odd part of the correlation function is zero ($a = 0$), and also

$$k\Delta_{13} \ F_{ijk} + \left(T_{R_3}^a\right)_{km}\left(T_{R_1}^a\right)_{i\ell} \ F_{\ell jm} = 0 \quad , \tag{20a}$$

$$k\Delta_{23}\ F_{ijk} + \left(T^a_{R_3}\right)_{km}\left(T^a_{R_2}\right)_{i\ell}\ F_{\ell jm} = 0 \quad , \tag{20b}$$

Using the current Ward identities (Eq. (0.13)), it is easy to show that eqs.(20a) and (20b) are equivalent. We therefore only consider Eq. (20b). After some straightforward algebra, it follows from Eq. (0.13) that

$$\left(T^a_{R_2}\right)_{j\ell}(T{R_3}^a)_{km}\ F_{i\ell m} = \frac{1}{2}\left(c_{R_1} - c_{R_2} - c_{R_3}\right)\ F_{ijk} \quad . \tag{0.21}$$

Consequently, if the fields Φ^2_R and Φ^3_R belong to degenerate representations, i.e., if $\Delta_2 = \frac{cR_2}{2k}$ and $\Delta_3 = \frac{cR_3}{2k}$, then $\Delta_1 = \frac{c_{R_1}}{2k}$. This proves the closure under operator-product expansion of the degenerate representations of the semidirect product of the superconformal and the super Kac-Moody algebra. Since any 3-point function of secondary fields is related via the superconformal and G-Ward identities to the 3-point function of the corresponding primary superfields our results apply to any 3-point function. This fact is important for the construction of a superstring theory on a group manifold, since it implies that the corresponding vertex operators form a closed algebra and the amplitudes factorize onto physical intermediate states.

When $c_A = k$, the representations of the super Kac-Moody algebra possess additional null states that are constructed out of the modes J^a_{-n}, ψ^a_{-r} $(n, r > 0)$. These states, however, are not highest-weight vectors of the semidirect product with the superconformal algebra.

It remains to consider the proper null highest-weight vectors of the Kac-Moody algebra, that is the ones obtained by the action of lowering operators of the Kac-Moody algebra only, on primary states. The operator algebra of those representations has been discussed in ref. [5] [3].

Combining the results of ref.[5] with ours, we have complete knowledge of the minimal system of representations of the super Wess-Zumino theory. In fact, the theory is exactly solvable in the sense that all the correlation functions satisfy a superequation of the form (0.19) and are therefore computable in principle. Below, we present an explicit evaluation of the 4-point function, which contains non-trivial information on the non-vanishing operator-product coefficients of the operator algebra. $OSP(2|1)$ invariance implies that the 4-point function is of the form

$$\begin{aligned} F_{ijk\ell} &\equiv \langle 0|\Phi^{R_1}_i(z_1,\theta_1)\,\Phi^{R_2}_j(z_2,\theta_2)\,\Phi^{R_3}_k(z_3,\theta_3)\,\Phi^{R_4}_l(z_4,\theta_4)\,|0\rangle \\ &= \sum_K A^K_{ijk\ell}\prod_{I<J}(z_{IJ})^{\gamma_{IJ}}\left[f_K(x) + y g_K(x)\right] \end{aligned} \tag{0.22}$$

where x, y are the two independent commuting combinations of the coordinates invariant under $OSP(2|1)$,

$$x = \frac{z_{12}z_{34}}{z_{13}z_2} \quad , \quad y = x + \frac{z_{14}z_{23}}{z_{13}z_{24}} - 1 \quad , \quad y^2 = 0 \quad , \tag{23a}$$

and

$$\gamma_{IJ} = \gamma_{JI} \quad , \quad \sum_{I\neq J}\gamma_{IJ} = -2\Delta_J \quad . \tag{23b}$$

[3] The selection rules derived in this case state that all non-integrable representations decouple.

Using the current Ward identities for the 4-point function, we can compute the group coefficient:

$$A^K_{ijk\ell} \sim \sum_{R,R',m,m'} \langle R_1, R_2, i, j|R, m\rangle\langle R', m'|R_3, R_4, k, l\rangle\langle R, R', m, m'|1_K, 0\rangle \quad . \tag{0.24}$$

where the index K labels the singlets in the product. The equation satisfied by the 4-point function can be derived in the same way as Eq. (0.19) (The variables here are the tilded ones (cf Eq. (0.17)):

$$\begin{aligned} &k\left[\sum_{i=1}^{3}\left(\frac{\partial}{\partial\theta_i} - \theta_i\frac{\partial}{\partial z_i}\right)\right] F_{ijk\ell} \\ &+\left(T^a_{R_4}\right)_{\ell m}\left(\frac{\theta_1}{z_1}\left(T^a_{R_1}\right)_{in} F_{njkm} + \frac{\theta_2}{z_2}\left(T^a_{R_2}\right)_{jn} F_{inkm} + \frac{\theta_3}{z_3}\left(T^a_{R_3}\right)_{kn} F_{ijnm}\right) = 0 \quad . \end{aligned} \tag{0.25}$$

We will present the solution to this equation for the simplest non-trivial case, G=SU(2), R_1, R_2, R_3, R_4, all being the fundamental of SU(2). Other cases do not require new tecniques but considerably more labor. There are two singlets in the product above,so we can write,

$$F_{ijk\ell} = F_1(x,y)\Delta_{ij}\Delta_{k\ell} + F_2(x,y)\Delta_{ik}\Delta_{j\ell} \tag{0.26}$$

(25) in this case is a two-by-two matrix equation. Using the identity,

$$\left(T^a_f\right)_{ij}\left(T^a_f\right)_{k\ell} = \frac{1}{2}\left[\Delta_{i\ell}\Delta_{jk} - \frac{1}{2}\Delta_{ij}\Delta_{k\ell}\right] \tag{0.27}$$

we can reduce it to two independent equations for F_1, F_2 which are of the hypergeometric type. Their solutions to lowest order in θ_i are,

$$F_1(x) = \left[\frac{(1-x)}{x^3}\right]^{\frac{1}{4k}} F\left(\frac{1}{2k}, -\frac{1}{2k}, 1 - \frac{1}{k,x}\right) \tag{28a}$$

$$F_2(x) = [x(1-x)]^{\frac{1}{4k}} F\left(\frac{3}{4k}, \frac{1}{4k}, 1 + \frac{1}{k}, x\right) \tag{28b}$$

It is straightforward to put back the $theta$-dependence and to normalize it correctly by factorizing it over two point functions.

The equation above has a very simple power-law solution in the special case where there is only one singlet contained in the product. We can define the constants α_{ij} as

$$(T^a_{R_4})_{lm}(T^a_{R_1})_{kn}F_{ijnm} = ka_{14}F_{ijkl} \quad , \tag{0.29}$$

and similarly for a_{24} and a_{34} where α_{14}, α_{24}, α_{34} are numbers. Using the Ward identity (13), we can show that $a_{14} + a_{24} + a_{34} = -\frac{c_{R_4}}{k} = -2\Delta_4$. Apart from the trivial solution, Eq. (0.25) has two other solutions

$$\gamma_{14} = a_{14} \quad , \quad g(x) = 0 \quad , \quad f(x) = Cx^{a_{34}-\gamma_{34}} \quad , \tag{30a}$$

and

$$\gamma_{14} = a_{14} - 1 \quad , \quad f(x) = 0 \quad , \quad g(x) = Cx^{a_{34}-\gamma_{34}-1} \quad , \tag{30b}$$

We can always eliminate γ_{12} by absorbing it into a redefinition of the function f or g. Then, in the first case, Eq. (26a), the exponents are determined to be:

$$\gamma_{14} = a_{14} \quad , \quad \gamma_{13} = -2\Delta_1 - a_{14} \quad , \quad \gamma_{34} = \Delta_1 + \Delta_2 - \Delta_3 - \Delta_4 \quad ,$$

$$\gamma_{24} = \Delta_3 - \Delta_2 - \Delta_4 - a_{14} \quad , \quad \gamma_{24} = \Delta_1 + \Delta_4 - \Delta_2 - \Delta_3 + a_{14} \quad . \tag{0.31}$$

In the second case, Eq. (26b), they are given by Eq. (0.27) if we make the substitution $a_{14} \to a_{14} - 1$.

The evaluation of higher correlation functions proceeds in a similar manner.

Ordinary Wess-Zumino models at their critical point describe the critical behavior of quantum chains with an arbitrary spin and continuous internal symmetry [3]. It would be interesting to see if some of these models are in fact supersymmetric, or if there are other critical systems that realize the semidirect product of the superconformal and the super Kac-Moody algebra.

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