Strings From Scratch

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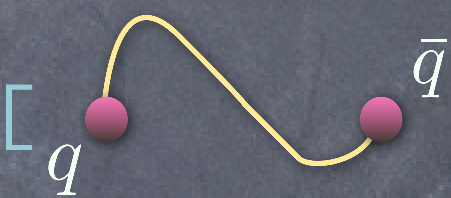
String Beginnings

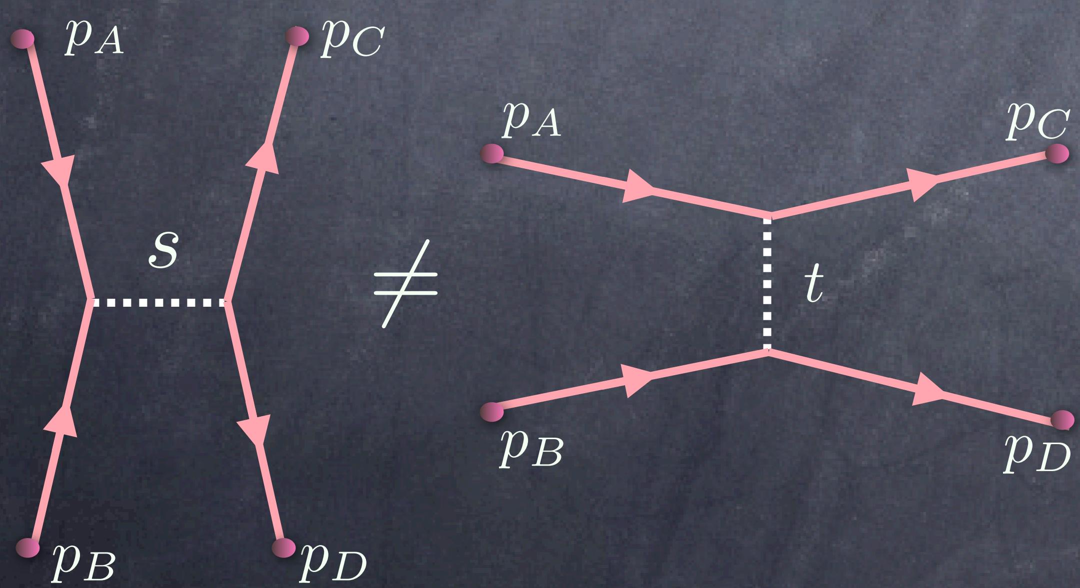
From Dual Resonance to Unification

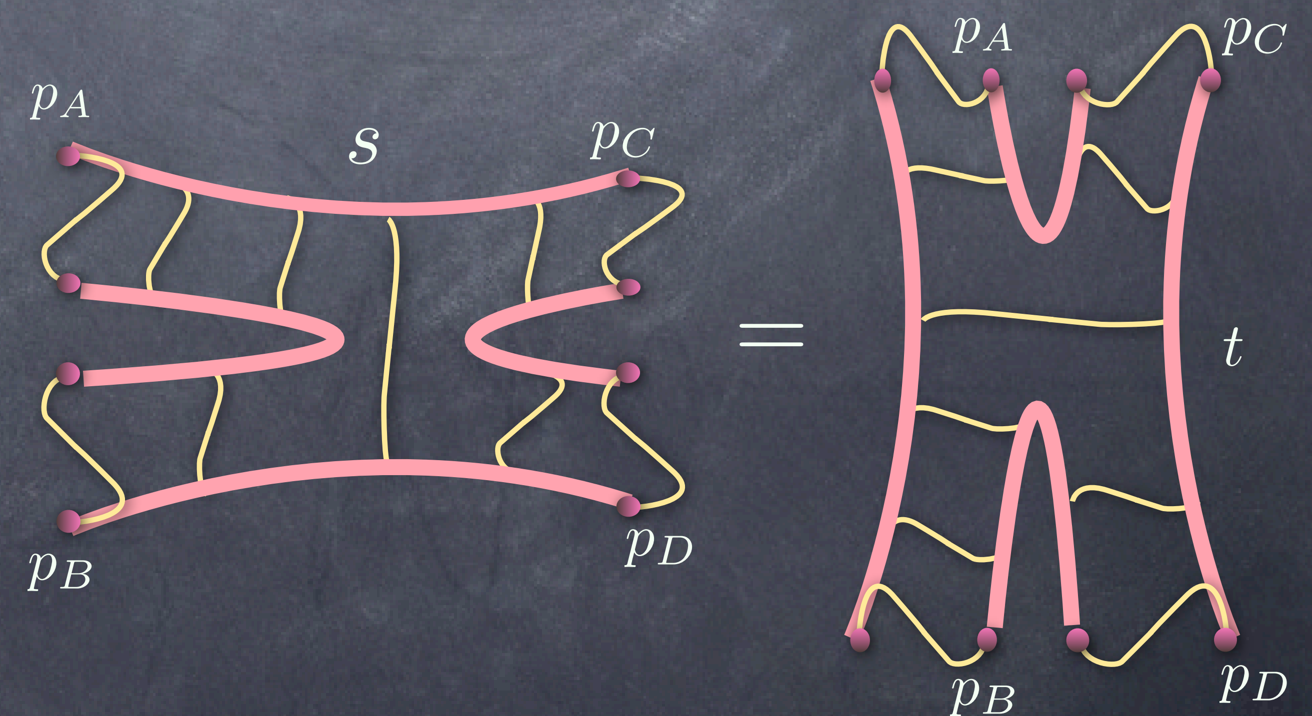
In the late 1960's, a phenomenological model of hadron physics emerges...

An s-t symmetry in resonance mediated hadron interactions is realized in the topological equivalence of extended object diagrams:

As an example, consider interacting mesons...

If Points [●]   If Strings []





String Beginnings

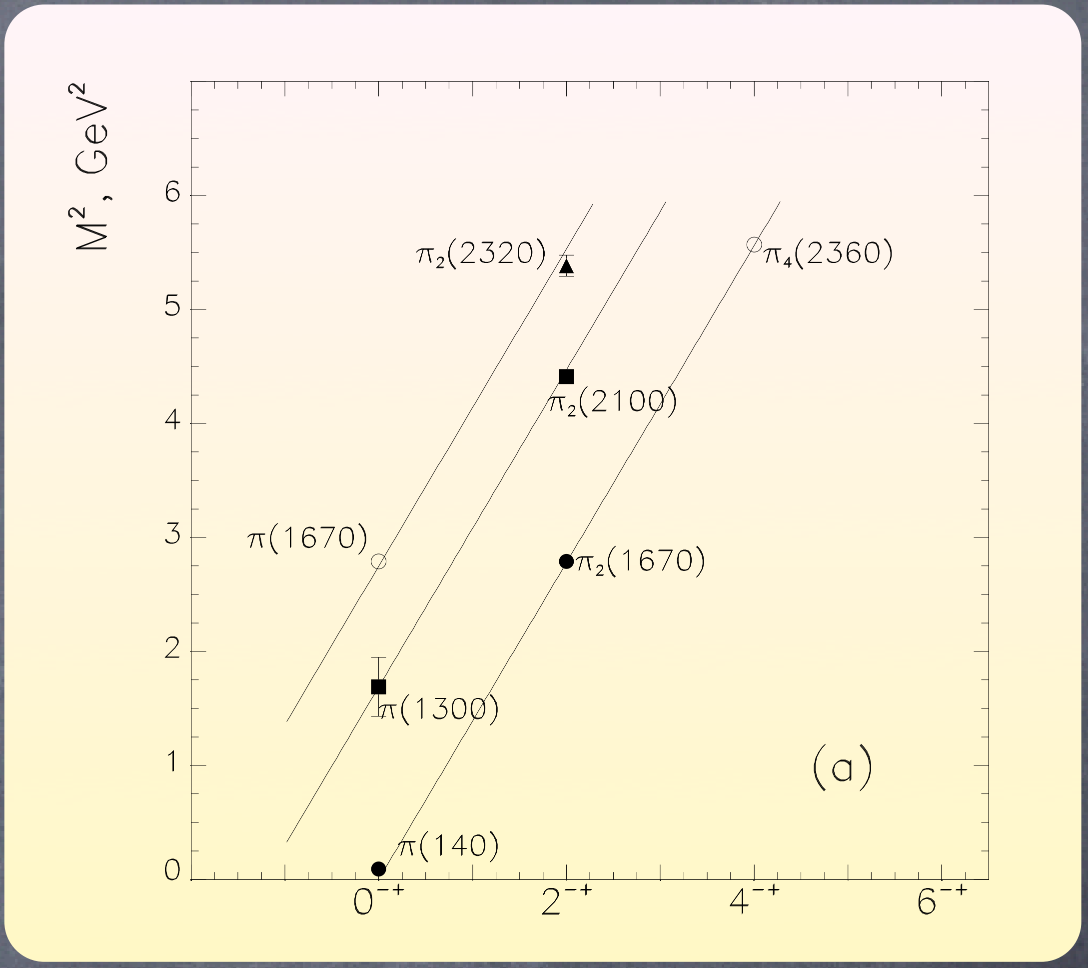
From Dual Resonance to Unification

In the late 1960's, a phenomenological model of hadron physics emerges...

The classical string model can account for the empirically observed linear rise in spin when plotted against M^2...

$$J(M) = \underbrace{\alpha_0}_{\text{Regge intercept}} + \underbrace{\alpha'}_{\text{Regge slope}} M^2$$

Hadronic resonances with the same (internal) quantum numbers but different masses lie on line of constant slope--"Regge Trajectories".



Relationship between s (mass squared) and J (spin) for pions, from [1]

A rotating relativistic string predicts this relationship between mass and spin!

$$J = \frac{1}{2\pi\tau} M^2 \quad \text{where} \quad \tau = \frac{1}{2\pi\alpha'} \approx 1\,\mathrm{GeV}^2 \quad \text{is the string tension}$$

String Beginnings

From Dual Resonance to Unification

- In the late 1960's, a phenomenological model of hadron physics emerges...
- ...And recedes a decade later with the arrival of QCD.
  - The "Lund String Model" still finds use in modern particle physics--e.g. contemporary p-p event generator PYTHIA uses it to fragment hadrons with reasonable success.
- Could a quantized theory of strings make sense?
  - Instead of treating the QCD flux tube as a string, make strings more (maximally) fundamental...

e	ν_e	u	c
μ	ν_μ	d	b
τ	ν_τ	s	t
W	Z	γ	g

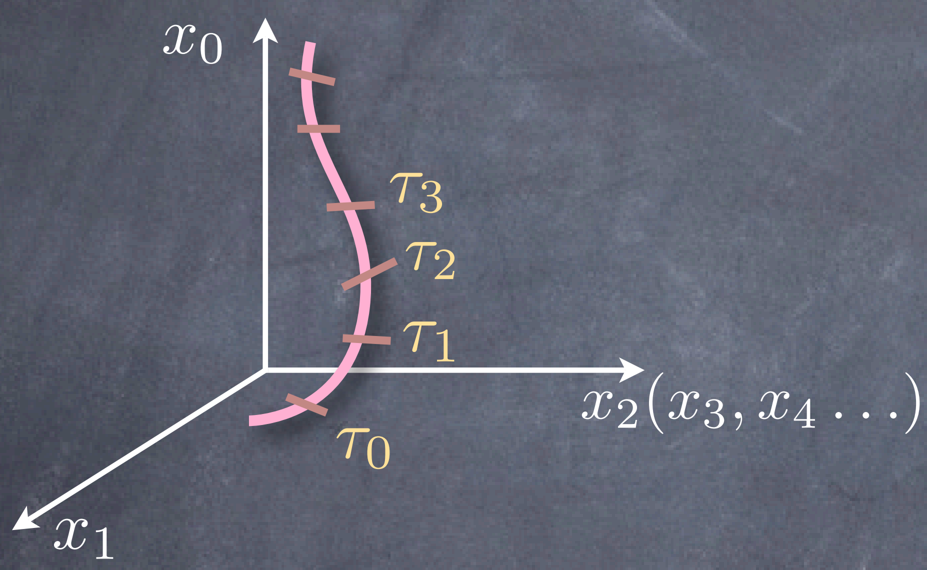
Modern string theory is born!

Bosonic String Basics

Where the action's at...

How should one describe the physics of strings?

Point Particle Primer:



A point particle traces out a curve in spacetime. Can parametrize $(\vec{x}(t) \longrightarrow x^\mu(\tau))$ the curve arbitrarily, must give the same physics.

For a massive particle, expect that the action is proportional to proper length of world-line:

$$S = -m \int ds = -m \int_\tau \sqrt{-\eta_{\mu\nu}\dot{x}^\mu \dot{x}^\nu}\, d\tau$$

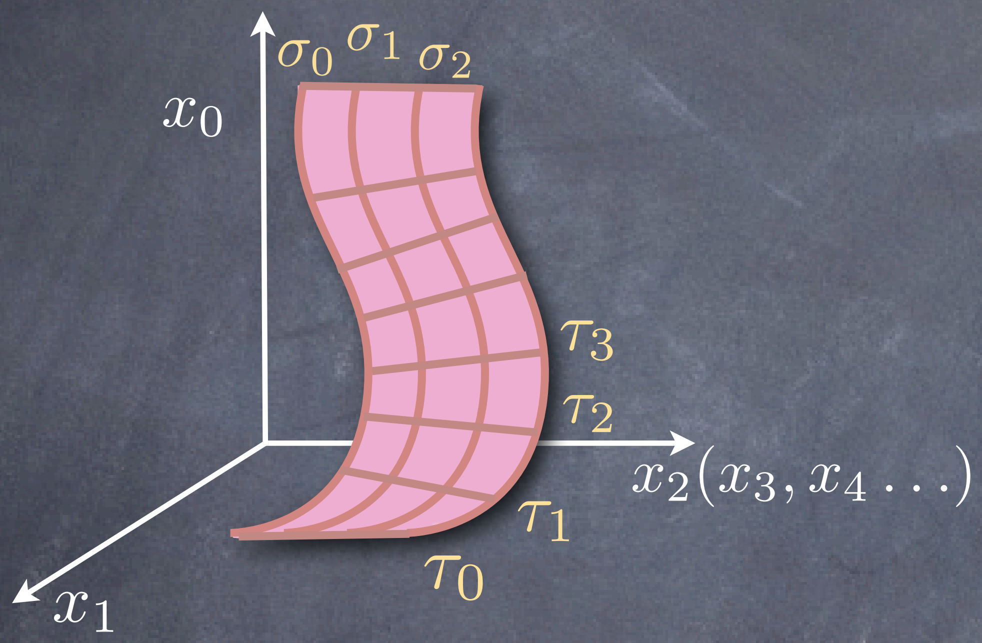
With metric (-,+,+,...,+) and $\dot{x} \equiv dx/d\tau$. Rightmost equality makes so-called "reparametrization invariance" explicit. For massless particles, must try harder. Introduce a metric on the world line, $h_{\tau\tau}(\tau)$ (e.g. [2]), then a classically equivalent action is

$$S = \frac{1}{2}\int d\tau \left(\frac{\dot{x}^\mu \dot{x}_\mu}{\sqrt{-h_{\tau\tau}}} - \sqrt{-h_{\tau\tau}}m^2 \right)$$

Bosonic String Basics

Where the action's at...

How should one describe the physics of strings?



A one dimensional string traces a surface in spacetime. Again the surface can have arbitrary parametrization. Surface described by embedding function

$$x^\mu(\tau, \sigma) \equiv X^\mu(\tau, \sigma)$$

With analogy to the relativistic point particle, expect an action proportional to area of string "world-sheet".

$$S_s \sim \int dA$$

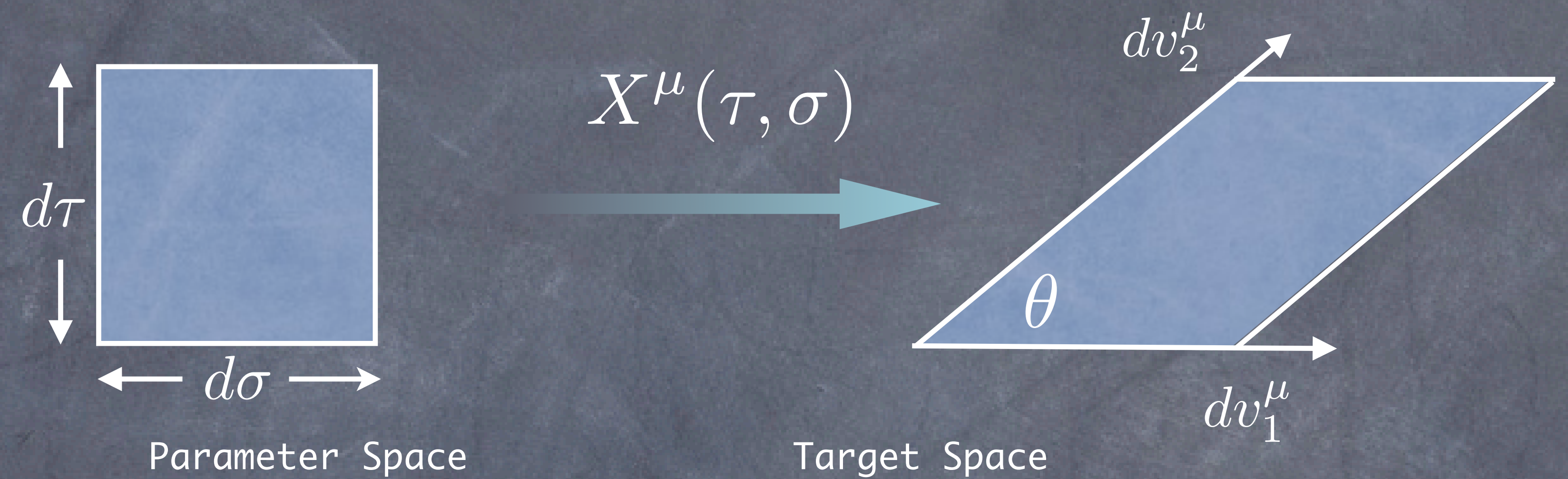
Need to find the Lorentz invariant area element...

Bosonic String Basics

Where the action's at...

How should one describe the physics of strings?

Need to find the Lorentz invariant area element...



The dv_α^μ span the transformed area element, and accordingly,

$$dv_i^\mu = \frac{\partial X^\mu}{\partial \alpha_i} d\alpha_i \qquad\qquad i = 1, 2\ ;\ \alpha_i = \tau, \sigma$$

so try

$$dA = \sqrt{(dv_1^\mu dv_{1_\mu})(dv_2^\mu dv_{2_\mu})}\,\sin\theta = \sqrt{(dv_1 \cdot dv_1)(dv_2 \cdot dv_2) - (dv_1 \cdot dv_2)^2}$$

Close, but turns out to be imaginary for points on the string world-sheet (reorder the difference).

Bosonic String Basics

Where the action's at...

How should one describe the physics of strings?

Using the expressions for the spanning set, find

$$S_s = -\frac{1}{2\pi\alpha'\hbar c^2}\int d\tau d\sigma\sqrt{(\dot{X}\cdot X')^2 - (\dot{X}\cdot\dot{X})(X'\cdot X')}$$

where the proportionality constant is constructed to give the correct units--bit more on this later.

Conventionally, this action is written in cleaner ("manifestly reparameterization invariant") form:

$$\boxed{S_{NG} = -\frac{1}{2\pi\alpha'}\int d\tau d\sigma\sqrt{-\det(\gamma_{ij})}}$$

Box is famed Nambu-Goto string action (natural units) written in terms of the induced metric on the world-sheet γ_{ij}, defined in the standard way...

$$-ds^2 = \eta_{\mu\nu}dX^\mu dX^\nu = \gamma_{ij}d\xi^i d\xi^j \qquad \text{where} \qquad \xi^1 = \tau \ , \ \xi^2 = \sigma$$

Bosonic String Basics

Where the action's at...

How should one describe the physics of strings?

Nambu-Goto action is intuitive, but hard to work with! In analogy to point particle, try to find classically equivalent action without derivatives in square root:

As before, add a metric $h_{\alpha\beta}(\tau,\sigma)$ on the world sheet, consider desirable symmetries, recoup NG equations of motion...

<u>Symmetry Wish List</u>

I. Poincare invariance in arbitrary-dimensional spacetime

$$X'^{\mu} = \Lambda^{\mu}{}_{\nu} X^{\nu} + b^{\mu} \qquad \delta h_{\alpha\beta} = 0$$

II. Diffeomorphism (reparametrization) invariance

$$X'^{\mu} = X^{\mu} \qquad h_{\alpha\beta} = \frac{\partial \xi'^{\gamma}}{\partial \xi^{\alpha}} \frac{\partial \xi'^{\delta}}{\xi^{\beta}} h'_{\gamma\delta}$$

III. Weyl invariance (world-sheet metric rescaling)

$$\delta X^{\mu} = 0 \qquad h_{\alpha\beta} \rightarrow \Omega^2(\tau,\sigma) h_{\alpha,\beta}$$

$$\boxed{S_p = -\frac{1}{4\pi\alpha'} \int d\tau d\sigma \sqrt{-\det(h_{\alpha\beta})}\, h^{\alpha\beta} \partial_\alpha X^\mu \partial_\beta X^\nu \eta_{\mu\nu}}$$

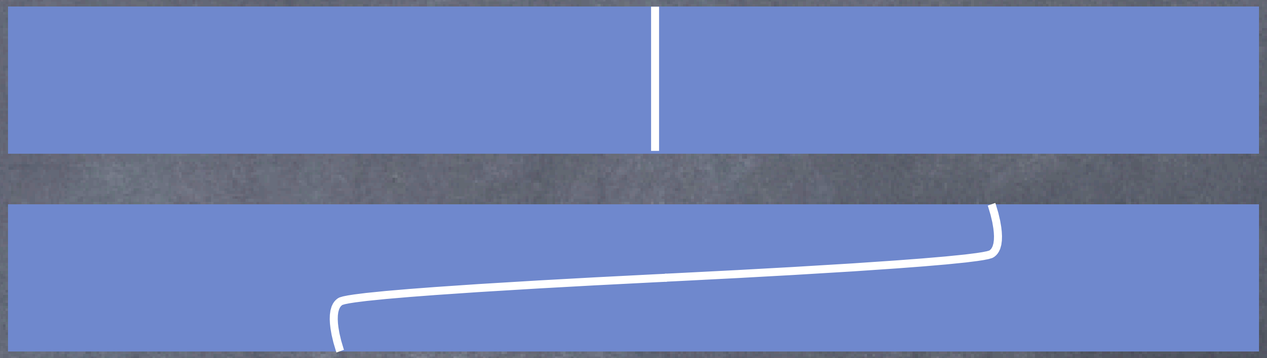
Bosonic String Basics

Where the action's at...

How should one describe the physics of strings?

S_P is Polyakov Action. Classically equivalent to S_{NG}, symmetry in all the right places, no square root over field derivatives (easier to quantize!)

In practice, exploit symmetries by fixing gauge:



Drawing a vertical line along the strip breaks translational symmetry. The line (akin to a gauge choice) keeps track of e.g. shear stresses but does not effect physical quantities. Idea from [3].

For topologically flat surfaces (no defects, etc.) convenient choice is conformal gauge...

$$h_{\alpha\beta} = \omega^2(\tau,\sigma)\eta_{\alpha\beta}$$

where $\omega(\tau,\sigma)$ is conformal factor, $\eta_{\alpha\beta}$ is flat-space Minkowski metric (-,+) in D=2. Then Polyakov action becomes

$$\boxed{S_p = -\frac{1}{4\pi\alpha'}\int d\tau d\sigma\, \eta^{\alpha\beta}\partial_\alpha X^\mu \partial_\beta X_\mu}$$

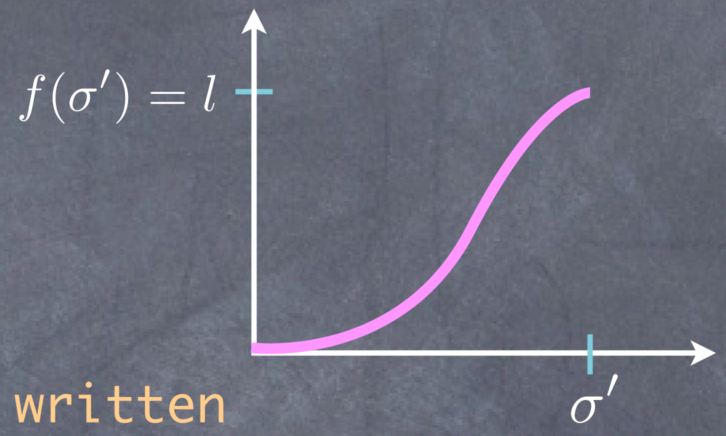
Bosonic String Basics

Where the action's at...

Some simple results from the string action:

Using the Nambu-Goto action, it is easy to study a relativistic stretched string. In static gauge, embedding function becomes

$$X^\mu(\tau, \sigma) = (c\tau, f(\sigma), 0, \ldots, 0)$$



and (with little algebra) action can be written

$$S_{NG} = -\frac{1}{2\pi\alpha'\hbar c}\int_t dt\, \Delta f(\sigma) \qquad \text{where} \qquad \Delta f(\sigma) = f(\sigma') - f(0) = l$$

Static strings have no kinetic energy, integrand (Lagrangian) is just opposite of potential energy, i.e.

$$U = \frac{1}{2\pi\alpha'\hbar c} l = T \cdot l$$

Energy per length is a force, just the string tension!

Bosonic String Basics

Where the action's at...

Some simple results from the string action:

Now a relationship between constants in the action and the string tension:

$$\alpha' = \frac{1}{2\pi T \hbar c}$$

With new dimensionful parameter, easy to create a characteristic string length from fundamental constants...

$$[L]^2 = \frac{[E][T][L]}{[F][T]} \quad \text{so} \quad l_s = \sqrt{\frac{\hbar c}{T}}$$

or better yet,

$$\boxed{l_s = \hbar c \sqrt{\alpha'}}$$

Bosonic String Basics

Where the action's at...

Some simple results from the string action:

Using the Polyakov action in the conformal gauge, easy to obtain the string equations of motion. From variation of fields,

$$-\eta^{\alpha\beta}\partial_\alpha\partial_\beta X^\mu = \ddot{X}^\mu - X''^\mu = 0 \qquad \text{(wave equation!)}$$

and from the variation of the metric, two constraints:

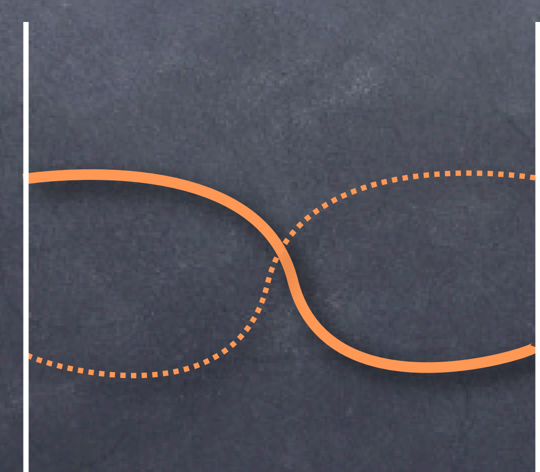
$$\dot{X}^\mu \cdot X'_\mu = 0 \qquad \text{and} \qquad \dot{X}^2 + X'^2 = 0$$

As always, a boundary term must vanish as well. If string parametrized like $\sigma \in [0,\pi]$ term is

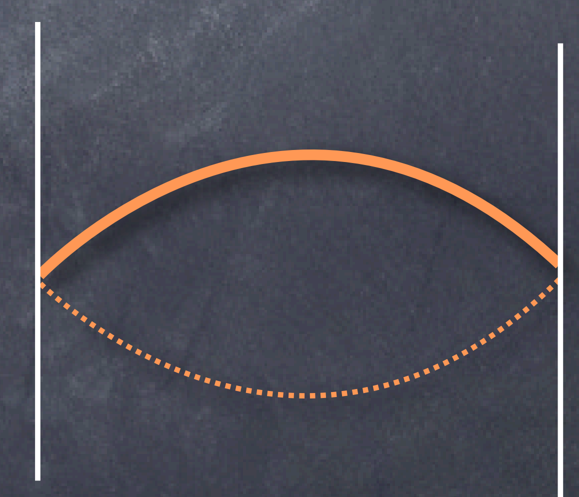
$$-\frac{1}{2\pi\alpha'}\int d\tau \delta X^\mu (X'_\mu)|_{\sigma=0}^{\sigma=\pi}$$

so possibilities are...

Closed (periodic) string



Neumann condition



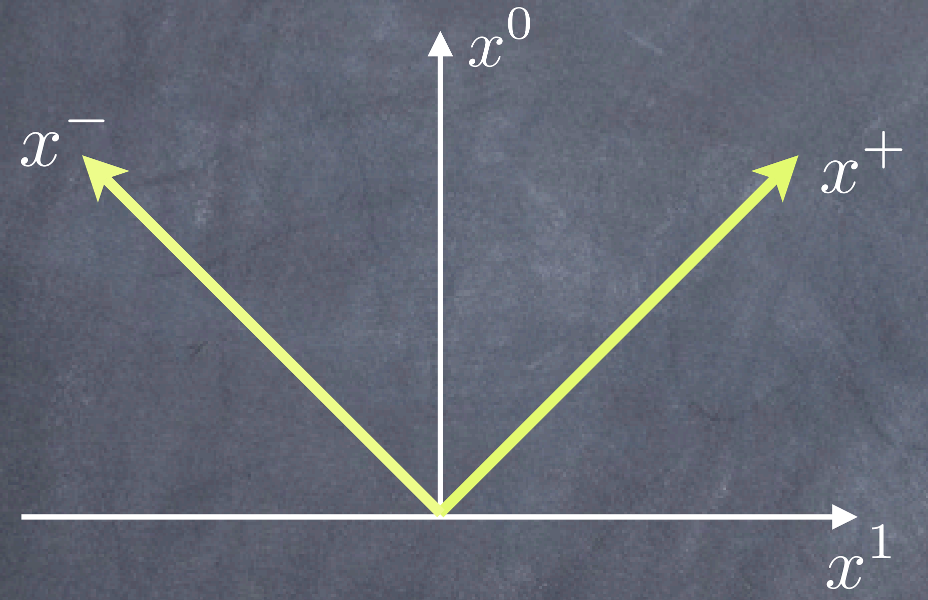
Dirichlet condition

Bosonic String Basics

Quantization Highlights

String theories are strange theories!

Already know we should start from Polyakov action, as fast track to physics break covariance and quantize in conformal + light-cone gauge (Fully specified, no negative norms).



Light cone coordinates are defined by axes that describe propagation of light:

$$x^+ = \frac{1}{\sqrt{2}}(x^0 + x^1)$$

$$x^- = \frac{1}{\sqrt{2}}(x^0 - x^1)$$

transverse coordinates remain unchanged. Covariance sacrificed because we ‘played favorites’ with x^0 and x^1. Flat Minkowski metric is now

$$\eta_{\mu\nu} = \begin{pmatrix} 0 & -1 & 0 & \dots & 0 \\ -1 & 0 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & 1 \end{pmatrix}$$

Bosonic String Basics

Quantization Highlights

String theories are strange theories!

Seek a 2 dimensional quantum theory on the surface of the string world-sheet. Action for transverse light-cone coordinates is

$$S = \frac{1}{4\pi\alpha'}\int d\tau d\sigma(\dot{X}_\perp \dot{X}_\perp - X'_\perp X'_\perp)$$

variation clearly returns wave equation as before. For Neumann strings, general solution is

$$X^\perp(\tau,\sigma) = q^\perp(\tau) + 2\sqrt{\alpha'}\sum_{n=1}^{\infty} q_n^\perp(\tau)\frac{\cos n\sigma}{\sqrt{n}}$$

inserting above into the action and integrating over σ (using orthogonality) leaves

$$S = \int d\tau \left[\frac{1}{4\alpha'}\dot{q}^\perp(\tau)\dot{q}^\perp(\tau) + \sum_{n=1}^{\infty}\left(\frac{1}{2n}\dot{q}_n^\perp(\tau)\dot{q}_n^\perp(\tau) - \frac{n}{2}q_n^\perp(\tau)q_n^\perp(\tau)\right)\right]$$

Which is just the action for a collection of oscillators!
(see, e.g. [4]).

Bosonic String Basics

Quantization Highlights

String theories are strange theories!

With oscillator description, once again on familiar territory. Can now write mode expansion for light-cone coordinates in terms of creation and annihilation operators, learn about state space.

(i) $$X^{\perp}(\tau,\sigma) = x_0^{\perp} + 2\alpha' p^{\perp}\tau + i\sqrt{2\alpha'}\sum_{n=1}^{\infty}\left(a_n^{\perp} e^{-in\tau} - a_n^{\perp\dagger} e^{in\tau}\right)\frac{\cos n\sigma}{\sqrt{n}}$$

(ii) $$X^{-}(\tau,\sigma) = x_0^{-} + \frac{1}{p^{+}} L_0^{\perp}\tau + \frac{i}{p^{+}}\sum_{n\neq 0} L_n^{\perp} e^{-in\tau}\frac{\cos n\sigma}{n}$$

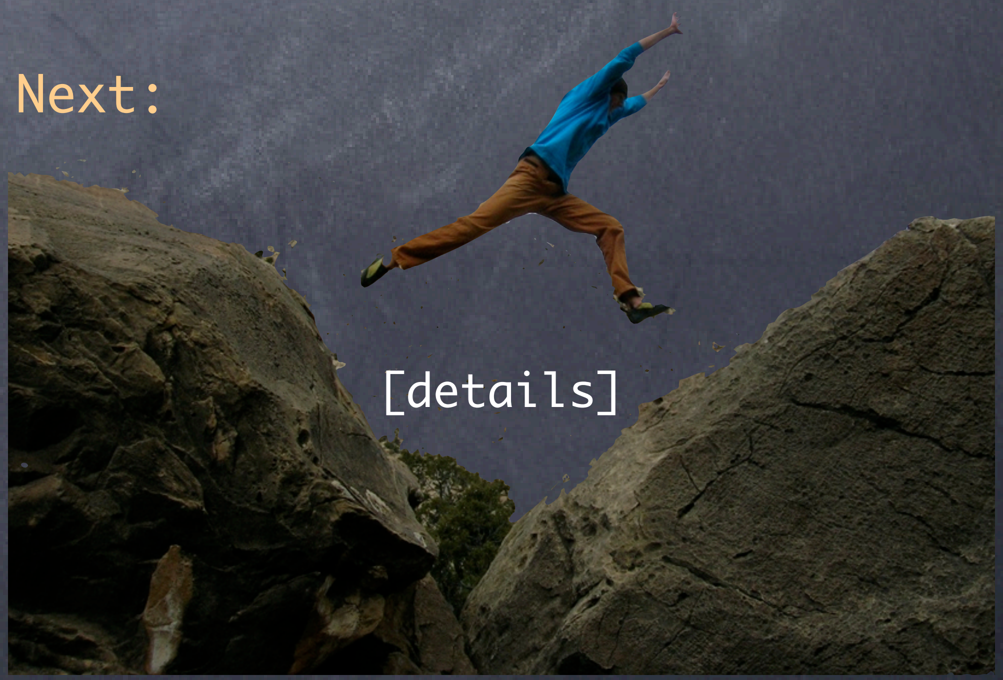
(iii) $$X^{+}(\tau,\sigma) = 2\alpha' p^{+}\tau$$

The $L_n^{\perp}$ that enter (ii) are the transverse Virasoro operators, which have an algebra

$$[L_m^{\perp}, L_n^{\perp}] = (m-n)L_{m+n}^{\perp} + \frac{D-2}{12}(m^3 - m)\delta_{m+n,0}$$

Central Charge

Next:



Bosonic String Basics

Quantization Highlights

String theories are strange theories!

Turns out that in definition of transverse Virasoro operators, there is an ordering issue for 0 mode operator. Expanding gives

$$L_0^\perp = \underbrace{\frac{1}{2}\alpha_0^\perp\alpha_0^\perp + \sum_{k=1}^{\infty}\alpha_{-k}^\perp\alpha_k^\perp}_{\text{redefined 0 mode operator}} + \underbrace{\frac{1}{2}(D-2)\sum_{k=1}^{\infty}k}_{\text{ordering constant } (a)}$$

Problem: 0 mode operator enters mass-squared operator, but looks like ordering constant diverges!

$$M^2 = -\eta_{\mu\nu}p^\mu p^\nu = \frac{1}{\alpha'}\left(L_0^\perp + a\right) - p^\perp p^\perp \quad \text{where} \quad a = \frac{1}{2}(D-2)\sum_{k=1}^{\infty}k$$

Not over yet--in maybe strangest analytic continuation ever concocted...

$$\sum_{k=1}^{\infty}k = \sum_{k=1}^{\infty}\frac{1}{k^{-1}} = \zeta(-1) = -\frac{1}{12}$$

can do this other ways, too, still get

$$\boxed{a = -\frac{1}{24}(D-2)}$$

Bosonic String Basics

Quantization Highlights

String theories are strange theories!

Enforcing Lorentz invariance in a theory of strings fixes the number of space time dimensions! Turns out one can only construct valid quantum Lorentz generators if...

$$\boxed{D = 26 \quad \text{and accordingly} \quad a = -1}$$

(Critical conditions)

could say that string theory predicts dimensionality of space it describes! Can do similar thing in superstring theory (with fermions), get famous critical dimension D = 10.

Now easy to write down the open string spectrum, defining a (normal ordered) transverse number operator

$$N^\perp \equiv \sum_{n=1}^{\infty} n a_n^{\perp\dagger} a_n^\perp = L_0^\perp - \alpha' p^\perp p^\perp$$

as obvious, simply counts total mode number of creation operators on given basis state e.g.

$$N^\perp a_4^{\perp\dagger}|0\rangle = \left[N^\perp, a_4^{\perp\dagger}\right]|0\rangle + \cancelto{0}{a_4^{\perp\dagger} N^\perp|0\rangle} = 4\, a_4^{\perp\dagger}|0\rangle$$

Bosonic String Basics

Quantization Highlights

String theories are strange theories!

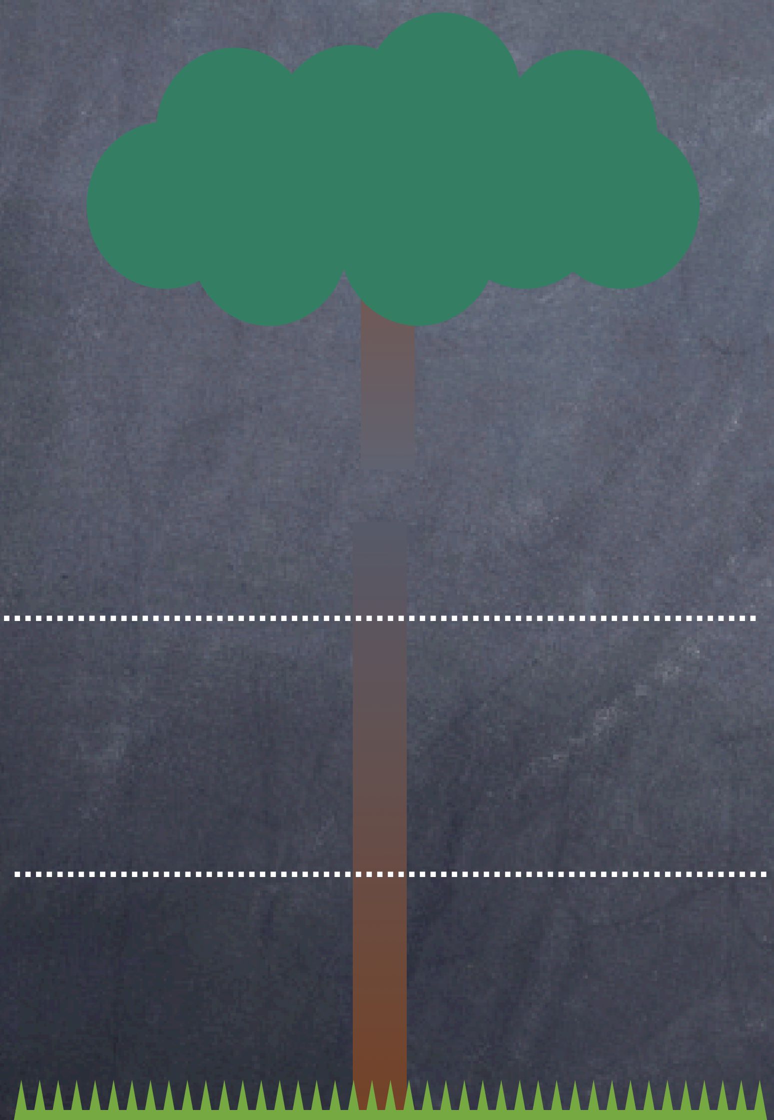
Hit states with mass squared operator to construct spectrum:

$$M^2 = \frac{1}{l_s^2}\left(N^\perp - 1\right) \quad \text{on} \quad |N^\perp\rangle = \prod_{n=1}^{\infty}\prod_{T=2}^{25}(a_n^{T\dagger})^{N_{n,T}}|0\rangle$$

number of times each creation operator acts

transverse coordinates

An infinite number of states!



$N^\perp = 2$ $\quad M^2 = \frac{1}{\alpha'}$ Massive tensor (324 states)

$N^\perp = 1$ $\quad M^2 = 0$ Photon (24 states)

$N^\perp = 0$ $\quad M^2 = -\frac{1}{\alpha'}$ Tachyon (1 state)

[String Family Tree]

Bosonic String Basics

Untangling String Interactions

The description...

Parameter space string world-sheets provide natural canvas for describing interacting strings. Choose a gauge where parameter space coordinates have convenient physical interpretations:

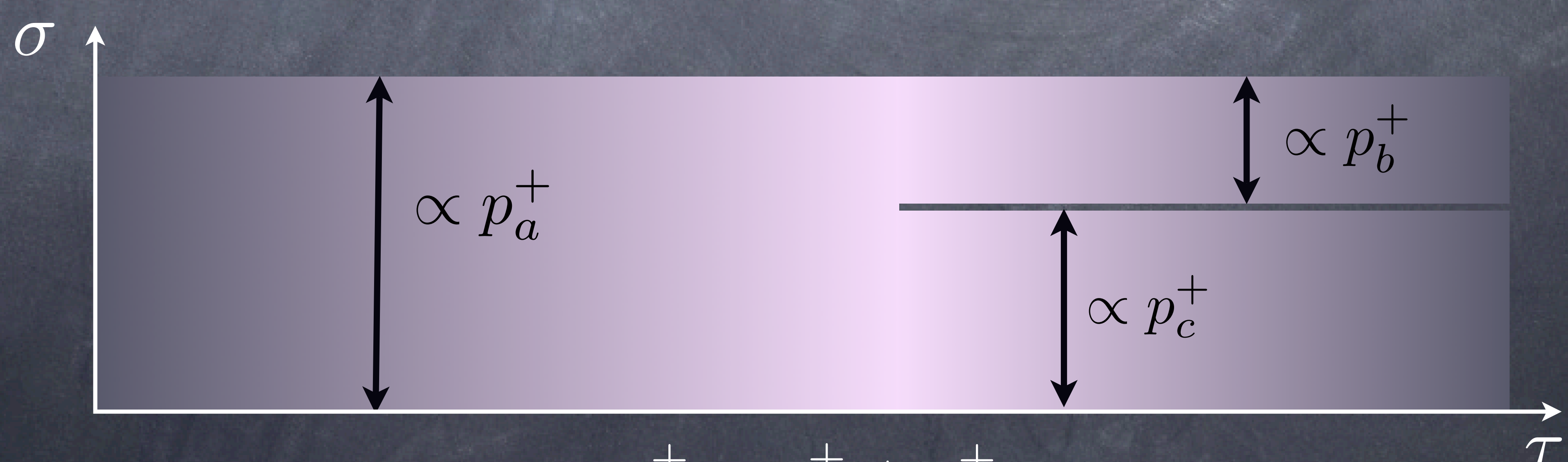
$$X^+ = \tau \qquad \text{and} \qquad \sigma \in \left[0, 2\pi\alpha' p^+\right]$$

world-sheet 'time'

string 'momentum' (conserved)!

Now string length in parameter space is useful...

[String Decay]



$$p_a^+ = p_b^+ + p_c^+$$

Bosonic String Basics

Untangling String Interactions

...And a beautiful connection

Question: How do we describe the physics of these (Weyl invariant) surfaces?

Answer: Wick rotate $h_{\alpha\beta}$ and identify the world-sheet with a Riemann surface!

Motivated by fact that Riemann surfaces are unchanged under conformal maps, world-sheets unchanged under Weyl transforms. Both locally scale distances.

Suggests string interactions described by conformally invariant field theory in 2D parameter space.

Instructions for computing string-string cross sections:

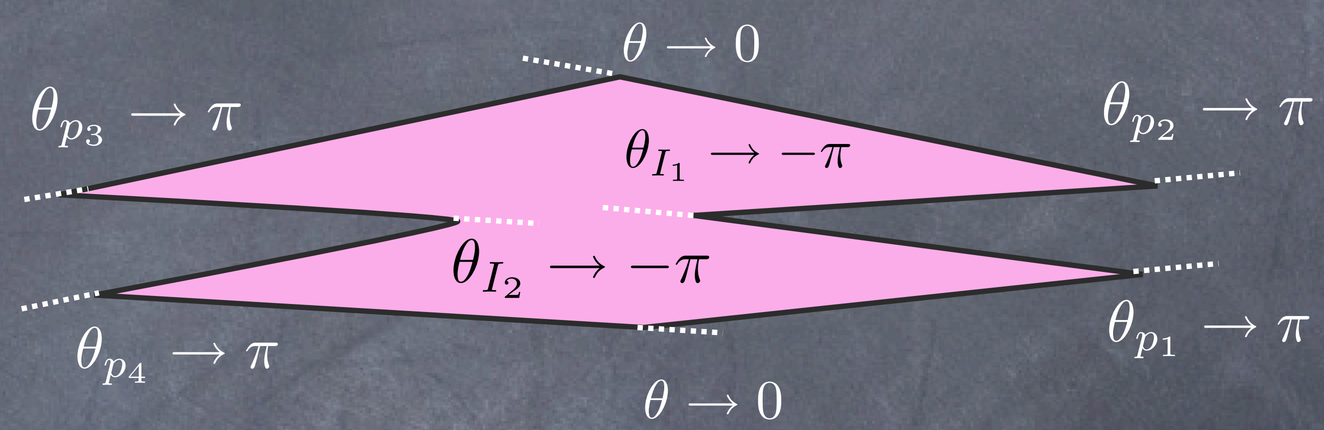
Bosonic String Basics

Untangling String Interactions

...And a beautiful connection

Map open strings to upper-half complex plane, plus point at infinity ($\bar{\mathbb{H}}$). Looks hard, but Schwarz-Christoffel save the day:

Consider world-sheets as degenerate polygons. Then write differential equation for function that maps points in $\bar{\mathbb{H}}$ to the polygon. Vertices go to real line...



$$\frac{dw}{dz} = A(z - x_1)^{-\frac{\theta_1}{\pi}}(z - x_2)^{-\frac{\theta_2}{\pi}} \ldots (z - x_{n-1})^{-\frac{\theta_{n-1}}{\pi}}$$

Where $z \in \bar{\mathbb{H}}$ and the “x” lie on the real line. For four string interaction above, map like $z(P_i) = 0, \lambda, 1, \infty$ and $z(I_i) = x_1, x_2$ so

$$\frac{dw}{dz} = A\frac{1}{z}(z - x_1)\frac{1}{z - \lambda}(z - x_2)\frac{1}{z - 1}$$

Note constraint on sum of angles allows omission of point at infinity.

Bosonic String Basics

Untangling String Interactions

...And a beautiful connection

We have an equation, but four unknowns. Use string “Feynman Rule” to produce additional constraints:

- For each string whose turning point is not mapped to infinity, add logarithm scaled by $\mp 2\alpha' p^+$ for σ that increase (decrease) over turning points [4].

for 4-string interaction,

$$w(z) = -2\alpha' p_1^+ \ln z - 2\alpha' p_2^+ \ln(z-\lambda) + 2\alpha' p_3^+ \ln(z-1)$$

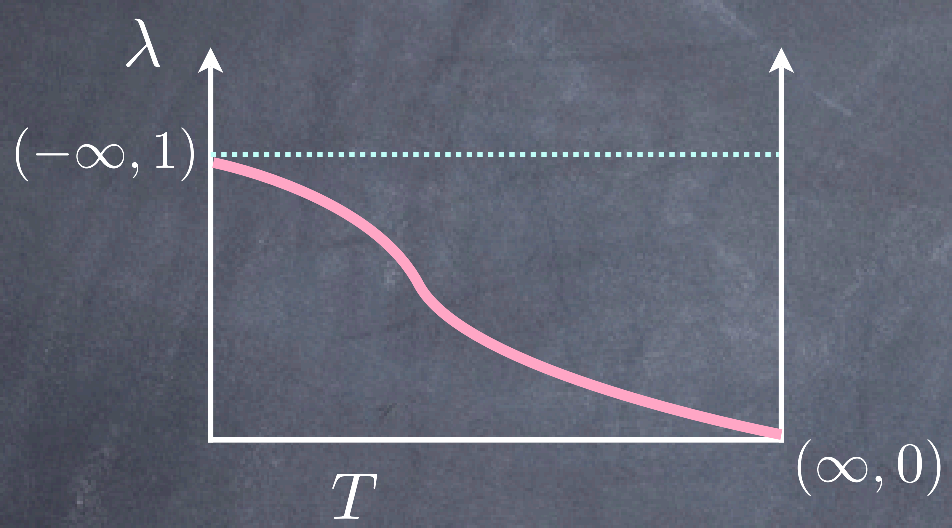
matching terms between above and previous formulations of differential equation, and using interaction time constraint fully determines all unknown parameters in mapping function. Mapped diagram looks like

Bosonic String Basics

Untangling String Interactions

...And a beautiful connection

A very important result: Using previous equations and constraints, can show



i.e. λ decreases monotonically on (0,1) over all times. For Riemann surface with four ordered punctures, moduli space is (0,1)--different $\lambda \in (0,1)$, (conformally) different surface--string diagrams produce moduli space of relevant Riemann surfaces!

Suggests intuitive method of handling string interaction diagrams. In QFT, sum all relevant diagrams; in ST, sum over all moduli?

$$\mathcal{M}_{ST} \overset{?}{\sim} g_s^2 \int_0^1 f(\lambda) d\lambda$$

Bosonic String Basics

Untangling String Interactions

...And a beautiful connection

Now right back where we started--for four open string tachyon scattering, Veneziano amplitude (with strings since 1960's) does the trick:

$$\mathcal{M}_{\mathcal{V}} = g_2^2 \int_0^1 d\lambda\, \lambda^{2\alpha'(k_1 \cdot k_2)}(1-\lambda)^{2\alpha'(k_2 \cdot k_3)} = g_s^2 \frac{\Gamma(-\alpha' s - 1)\Gamma(-\alpha' t - 1)}{\Gamma(-\alpha' s - \alpha' t)}$$

can obtain this amplitude (and others) in Polyakov path integral formalism with help from vertex operators [2]. Probe interaction structure by investigating amplitude poles...

Gamma function facts: No zeros, analytic continuation gives poles at $\alpha' s + 1 = n = 0, 1, 2, \ldots$ since poles at s signal existence of intermediate particle with squared mass equal to s,

$$\boxed{s = M^2 = \frac{1}{\alpha'}(n-1)}$$

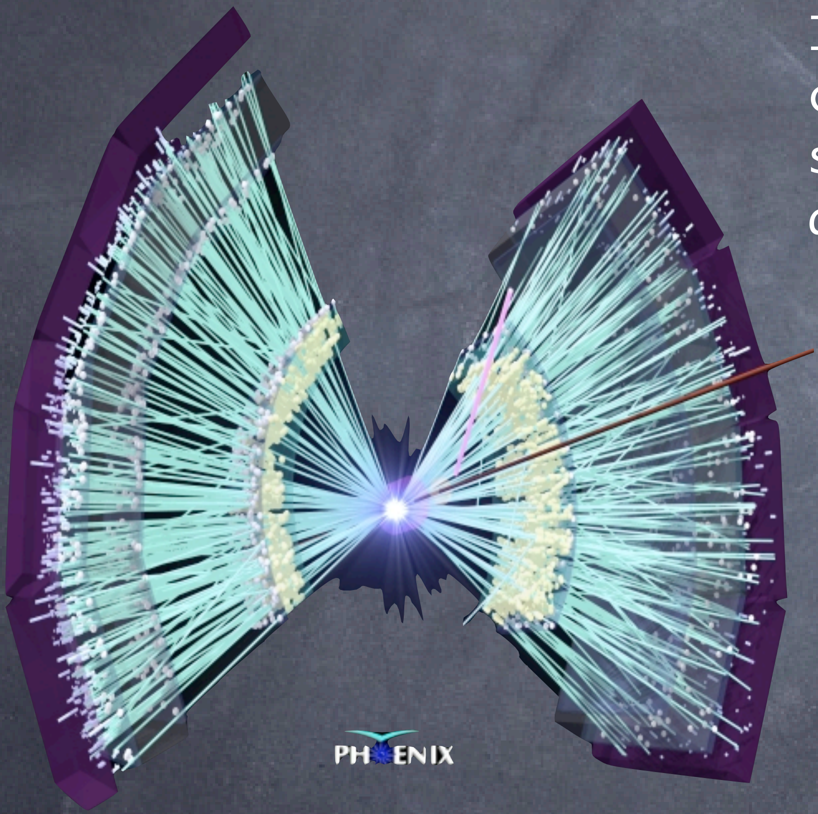
same spectrum as obtained for a quantized theory of relativistic strings!

Bosonic String Basics

Strings in the 21st Century

Heyyyyy Maldecena!

Many avenues for research in contemporary string theory. Here, one slide on my favorite...



In 1997 Juan Maldecena proposes an equivalence between conformal field theories (like $\mathcal{N}=4$ SYM) and a 10 dimensional string theory (AdS/CFT correspondence). Exploiting the duality allows one to calculate in (e.g.) strongly coupled regimes.

Measurements from the Relativistic Heavy Ion Collider suggest that the produced matter in $\sqrt{s_{NN}} = 200$ GeV Au+Au collisions is a strongly coupled "plasma" of quarks and gluons (QGP):

D. Son et. al. [5] show that there is a (universal) lower bound on the ratio of viscosity to entropy density for "strongly coupled" systems. RHIC data suggests QGP may be most perfect fluid ever created!

S. Gubser [6] computes drag force on a quark traversing a super Yang-Mills thermal plasma. Calculation provides string theory analog of jet quenching phenomenon observed in heavy ion collisions.

K. Rajagopal et. al. [7] study quarkonia propagation in a strongly coupled plasma. Also predict a value for the stopping power of the produced matter, not unreasonable!

QCD is not a CFT--can this duality produce meaningful results?

Bosonic String Basics

Conclusions, Comments...Concerns?

Where we've been, where we're going

- String theories have been around 40 years--initially formulated as description of strong interactions, now a description of all interactions.
- A logical set of symmetries leads to an action full of surprises.
- Scattering amplitudes in string theory are supported by an elegant mathematical framework.
- Curious gauge/string duals raise questions about what it means to be strongly coupled, toy model makes interesting predictions for hot QCD matter.
- Characteristic string length is very small and to date no testable predictions formulated. Yet as John Schwarz has said, the theory does predict gravity!
- Although sophistication of these slides is modest, understanding material presented here greatly increases accessibility of string literature.

References

Below is a list of works cited in the preceding slides, along with other resources used in preparing this presentation. Particularly interesting or lucid references have been highlighted:

[1] A.V Anisovich et al. hep-ph/0003113

[2] J. Polchinski, String Theory; Cambridge University Press (2001)

[3] "Gauge Fixing", http://en.wikipedia.org/wiki/Ξ_gauge

[4] B. Zwiebach, A First Course in String Theory; Cambridge University Press (2004)

[5] P. Kovtun et. al. hep-th/0405231

[6] S.Gubser. hep-th/0605182

[7] H. Liu et. al. hep-ph/0605178

[8] E. D'Hoker and D.Freedman, TASI 2001 hep-th/0201253

[9] K. Becker et. al. String Theory and M-theory; Cambridge University Press (2007)

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