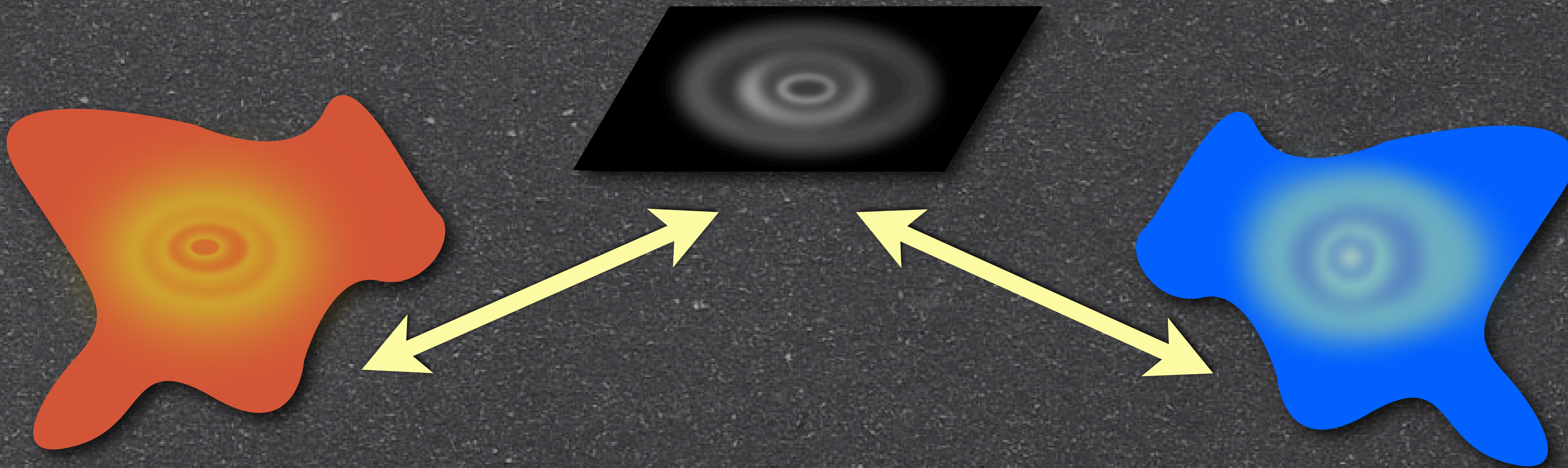


Applied AdS/CFT:

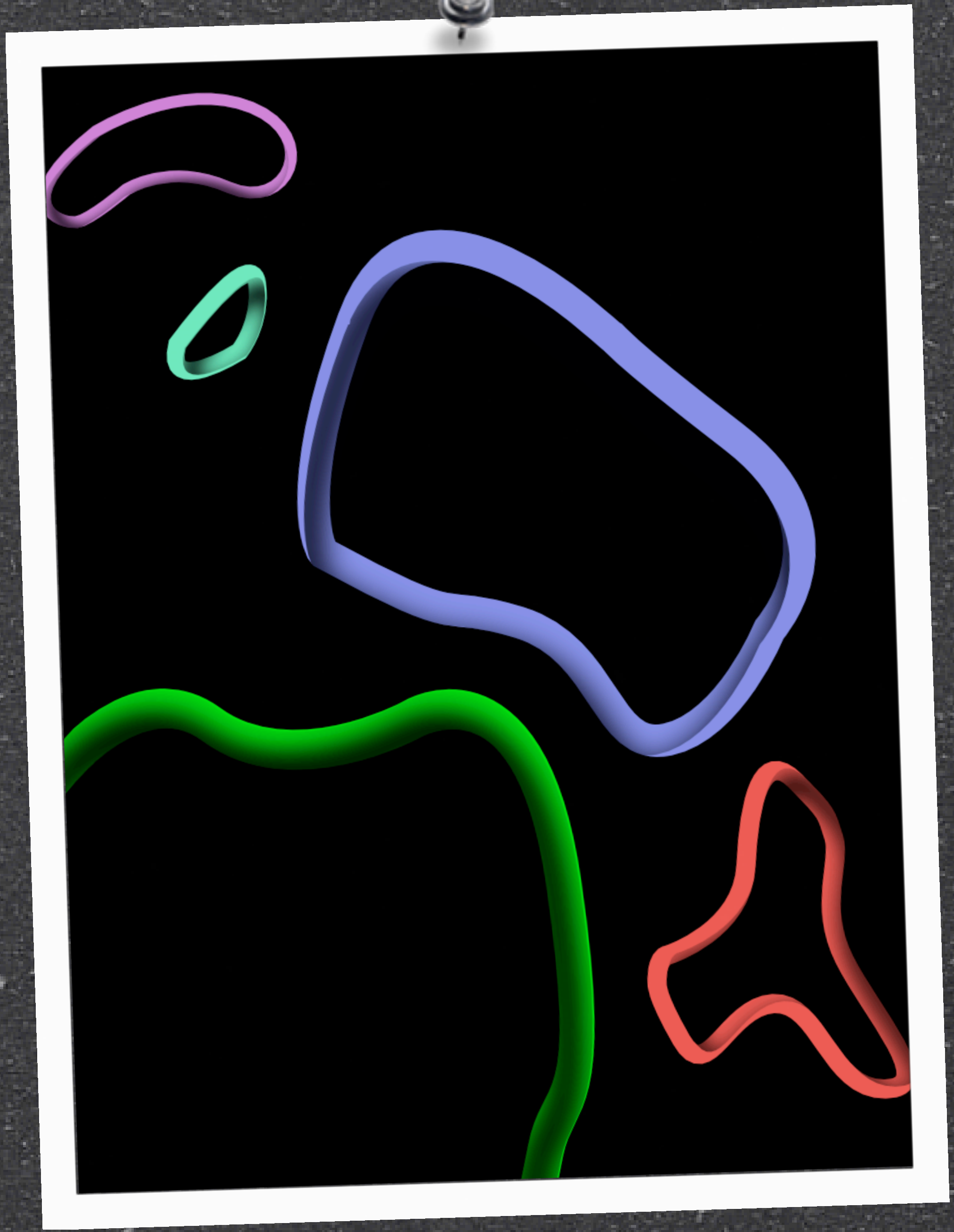
From hot quarks to condensed matters

Christopher Rosen

University of Colorado, Boulder

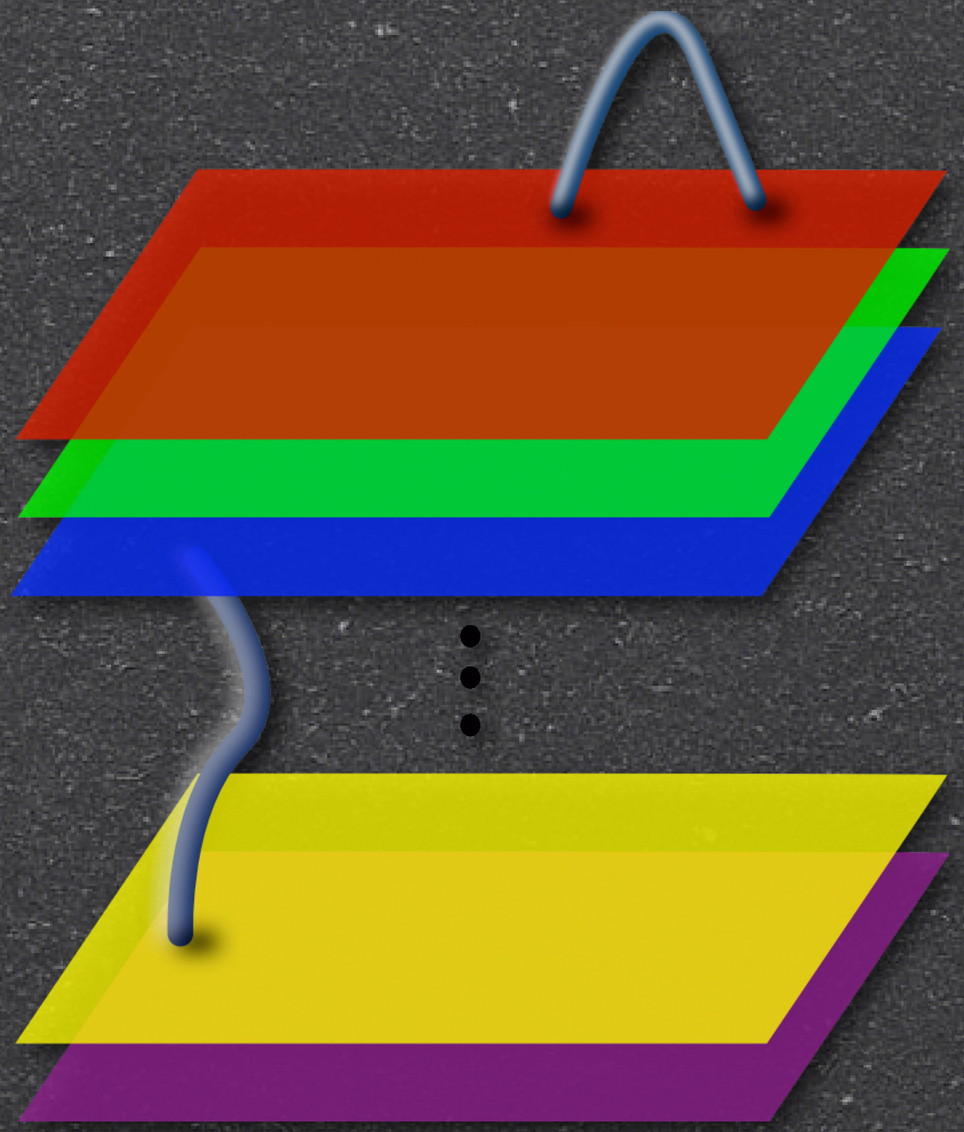
The Line Up

- What does AdS have to do with CFT?
- Strings and strong coupling in QCD
- Strings and strong coupling in CMT
- The Longview



Foundations

String theory contains strings, branes...



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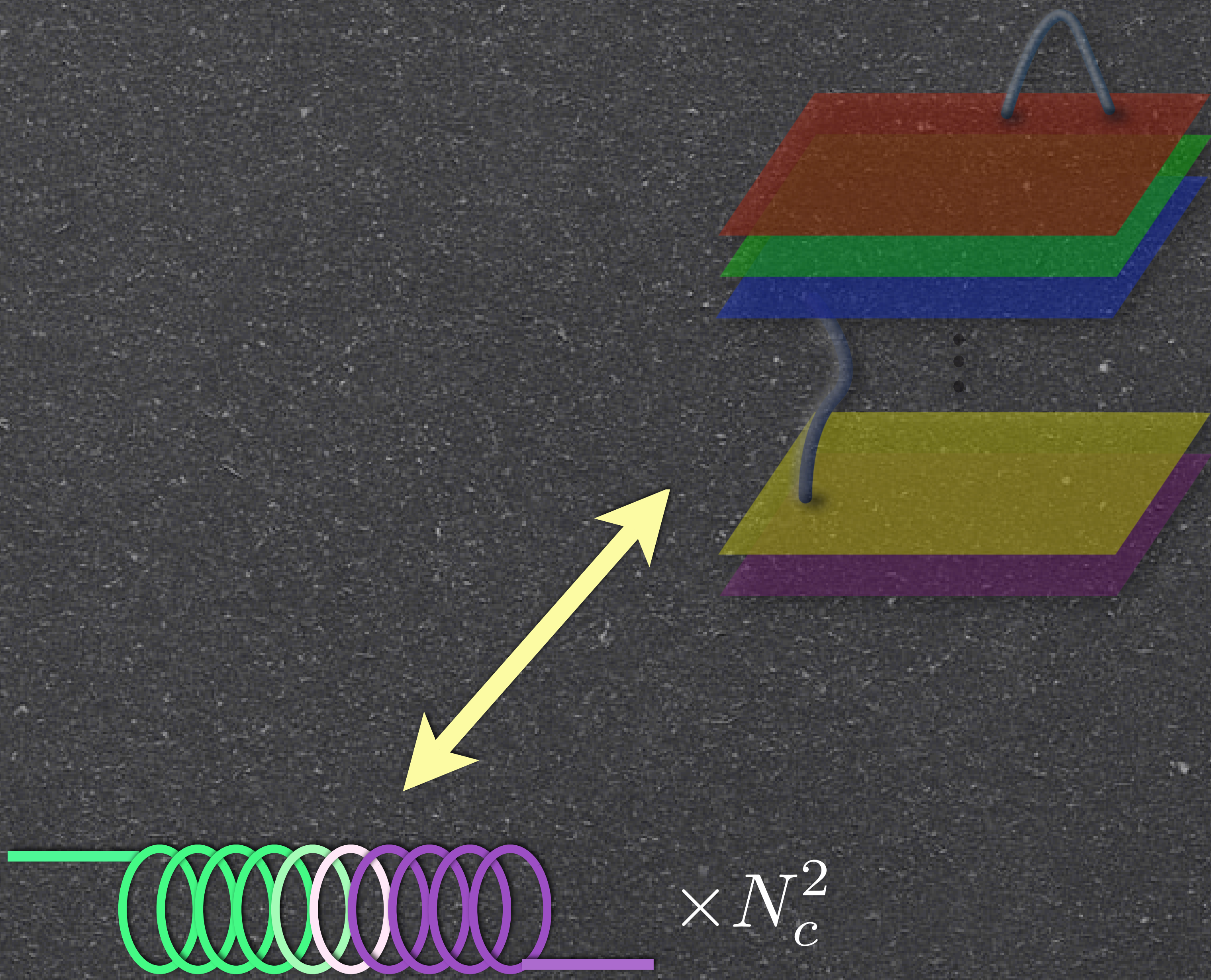
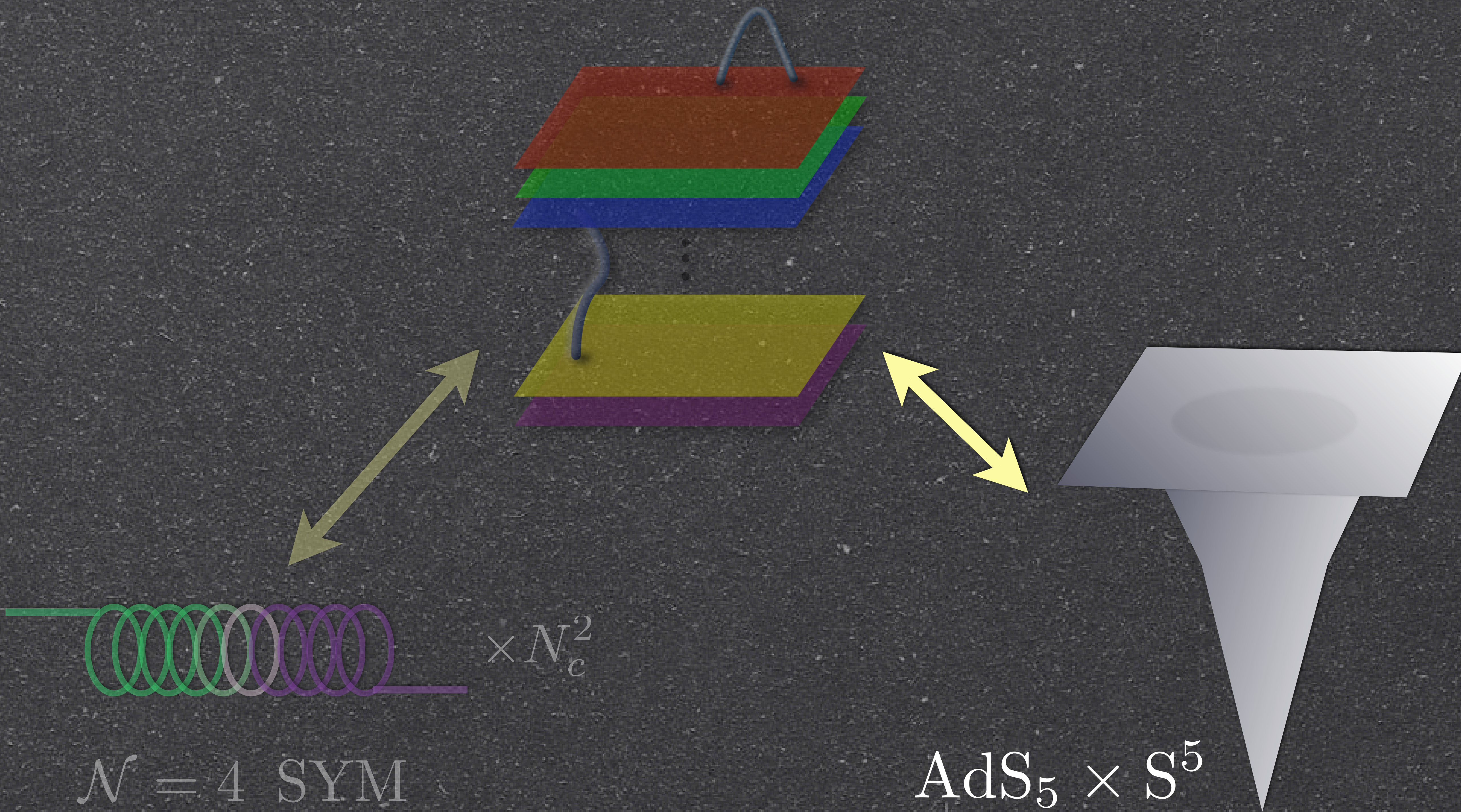


$\times N_c^2$

$\mathcal{N} = 4$ SYM

Foundations

String theory contains strings, branes...



Foundations

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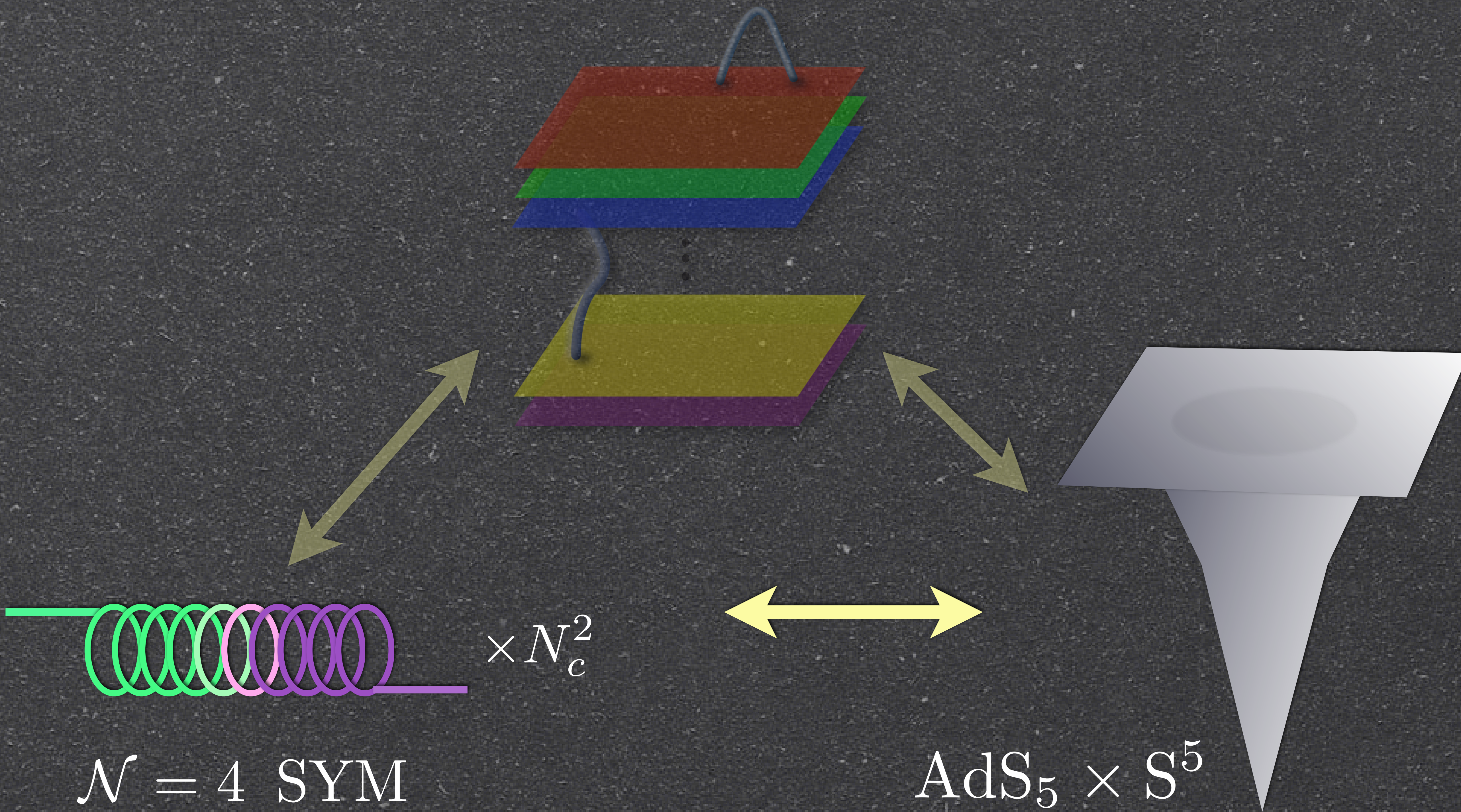
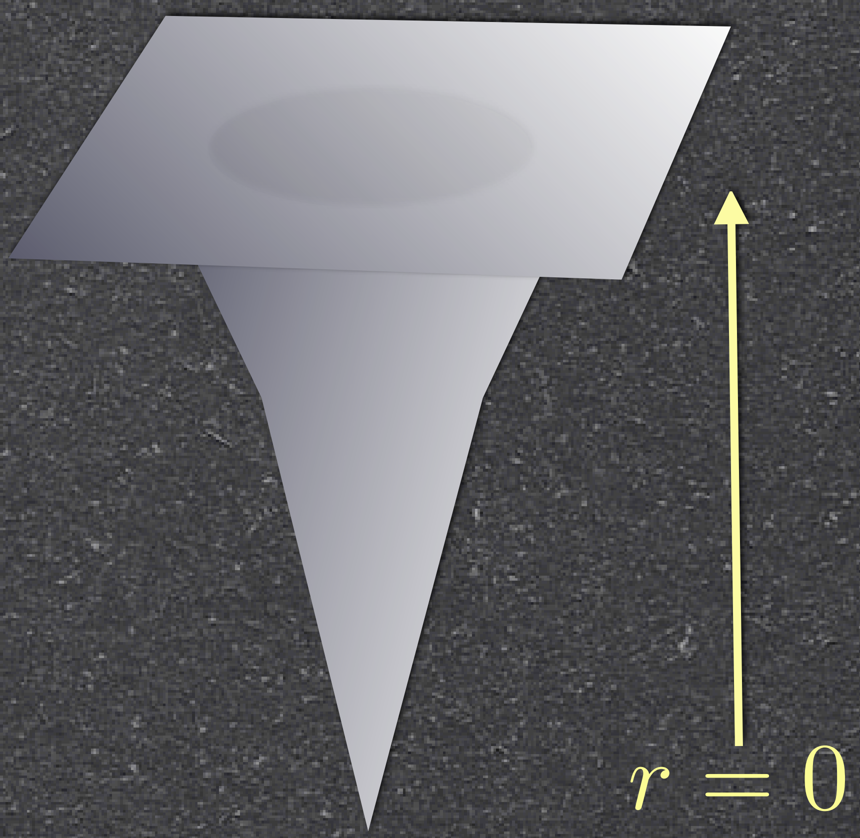


AdS & CFT

Why should Anti-de Sitter space help us study Conformal Field Theories?



The AdS metric:

$$ds^2 = \frac{r^2}{L^2}\left(-dt^2 + d\vec{x}^2\right) + \frac{L^2}{r^2}dr^2$$

An interesting isometry:

$$r \rightarrow \frac{r}{\lambda} \;\; \vec{x} \rightarrow \lambda\vec{x} \;\; t \rightarrow \lambda t$$

AdS & CFT

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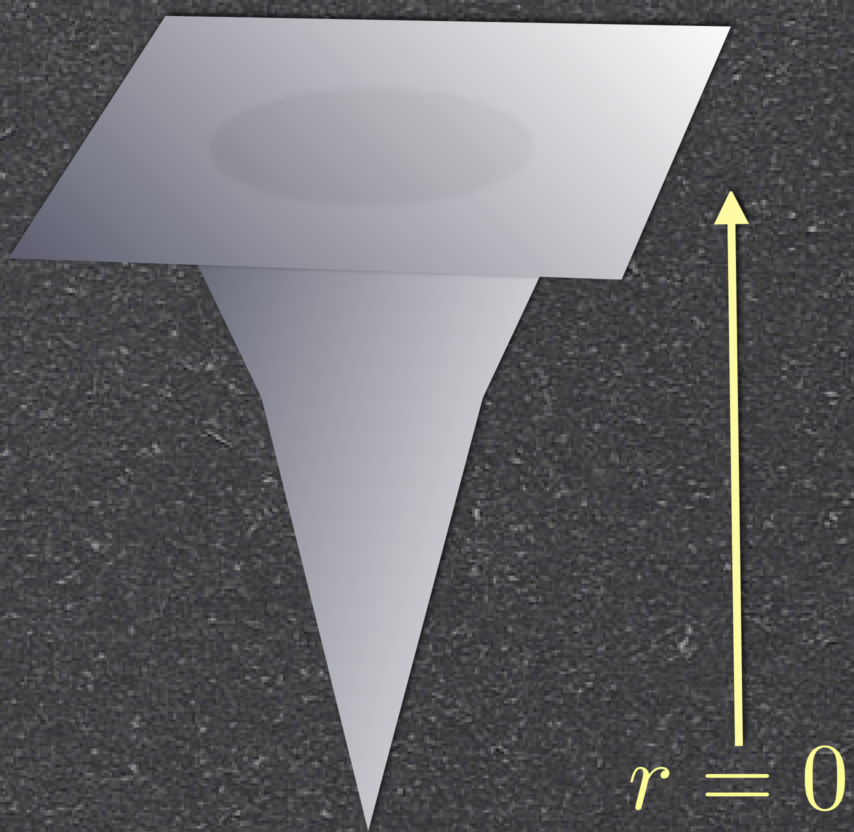

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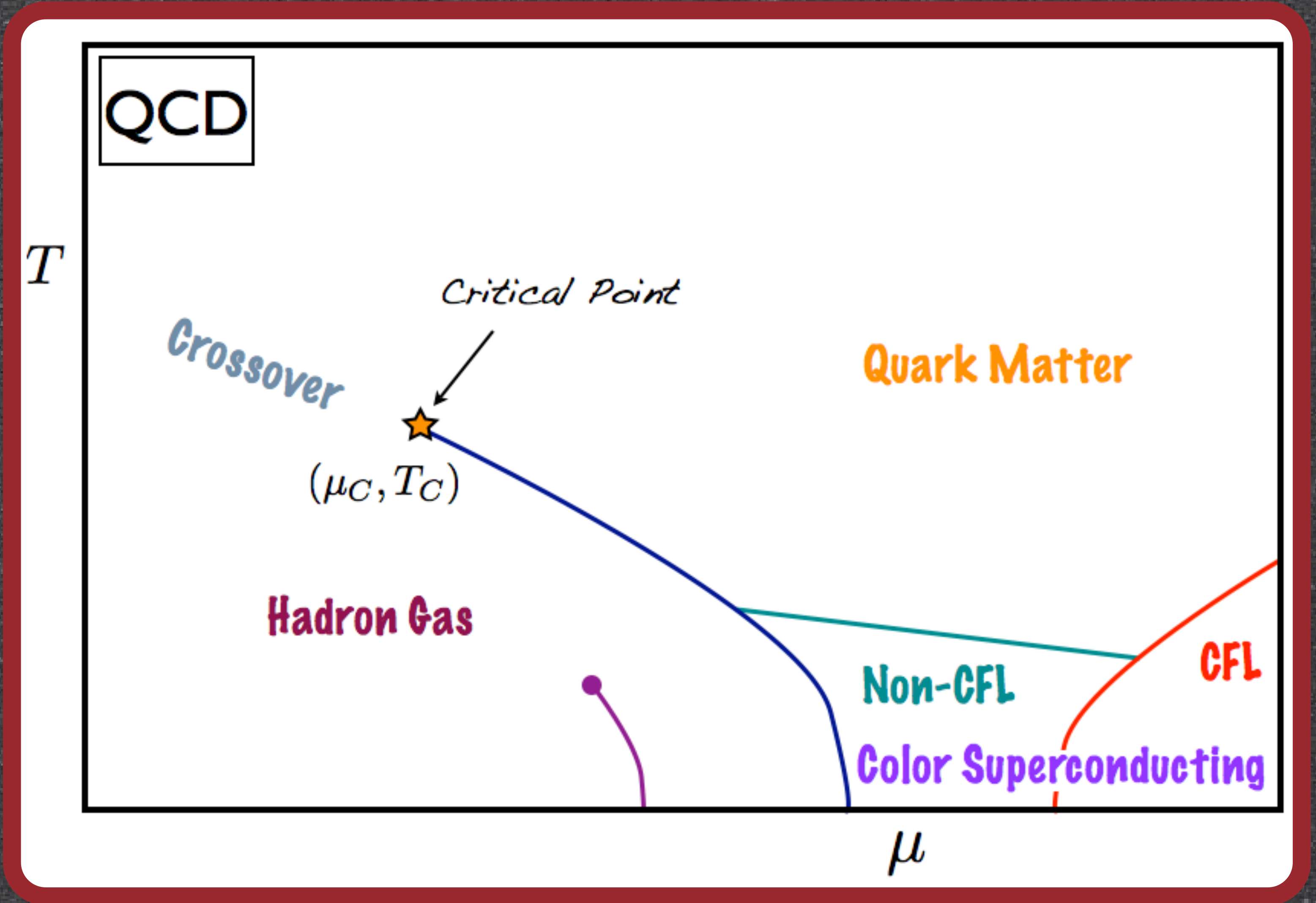
So this spacetime exhibits scale invariance and naturally associates r to an energy scale.

AdS & QCD

What do we want to know about QCD?

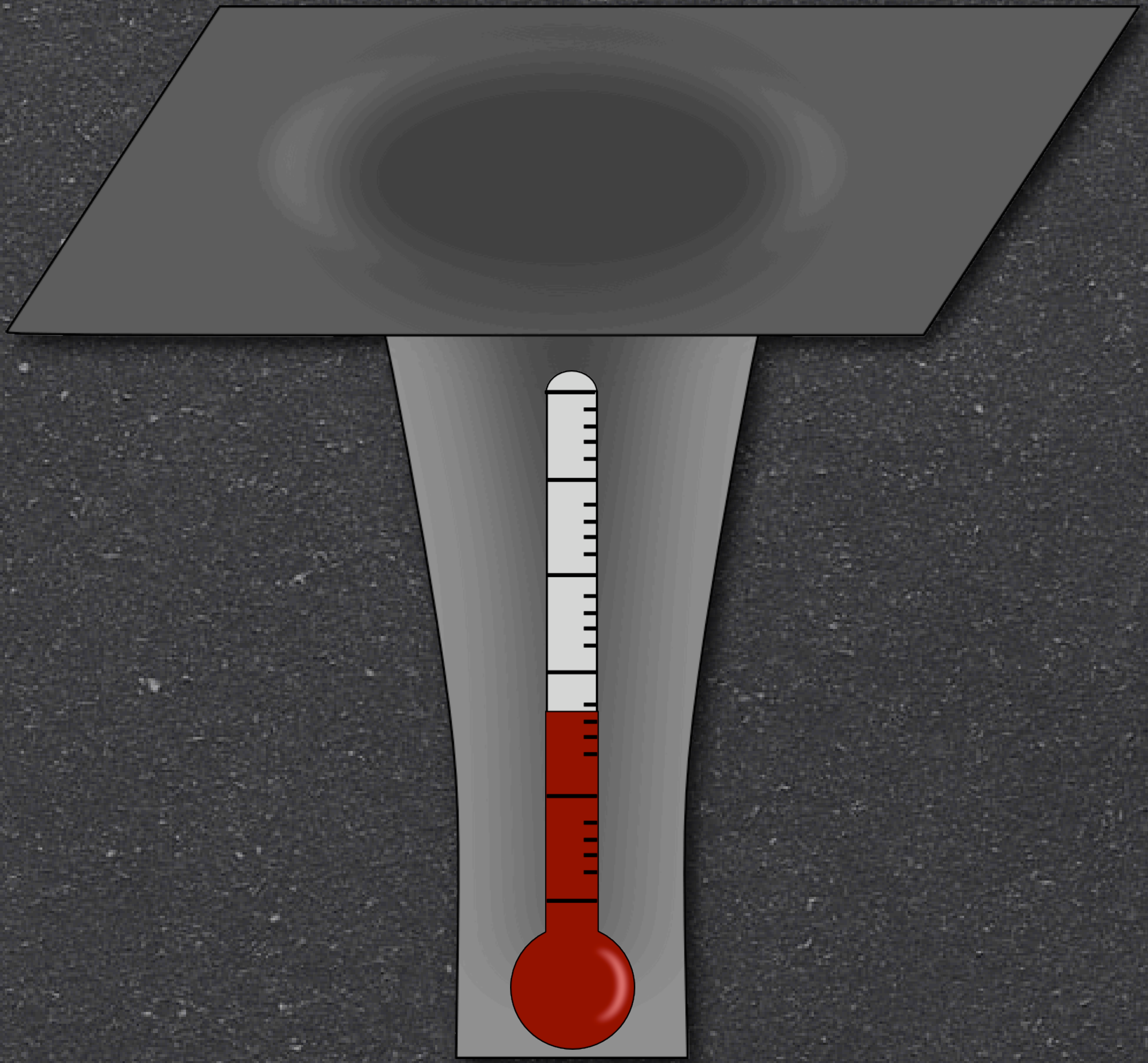
Goal: construct a dual to a “QCD-Like” theory that can be studied at finite temperature and chemical potential.



AdS & QCD

What gravity theory looks like hot QCD?

Need a temperature,
entropy,
hydrodynamics...



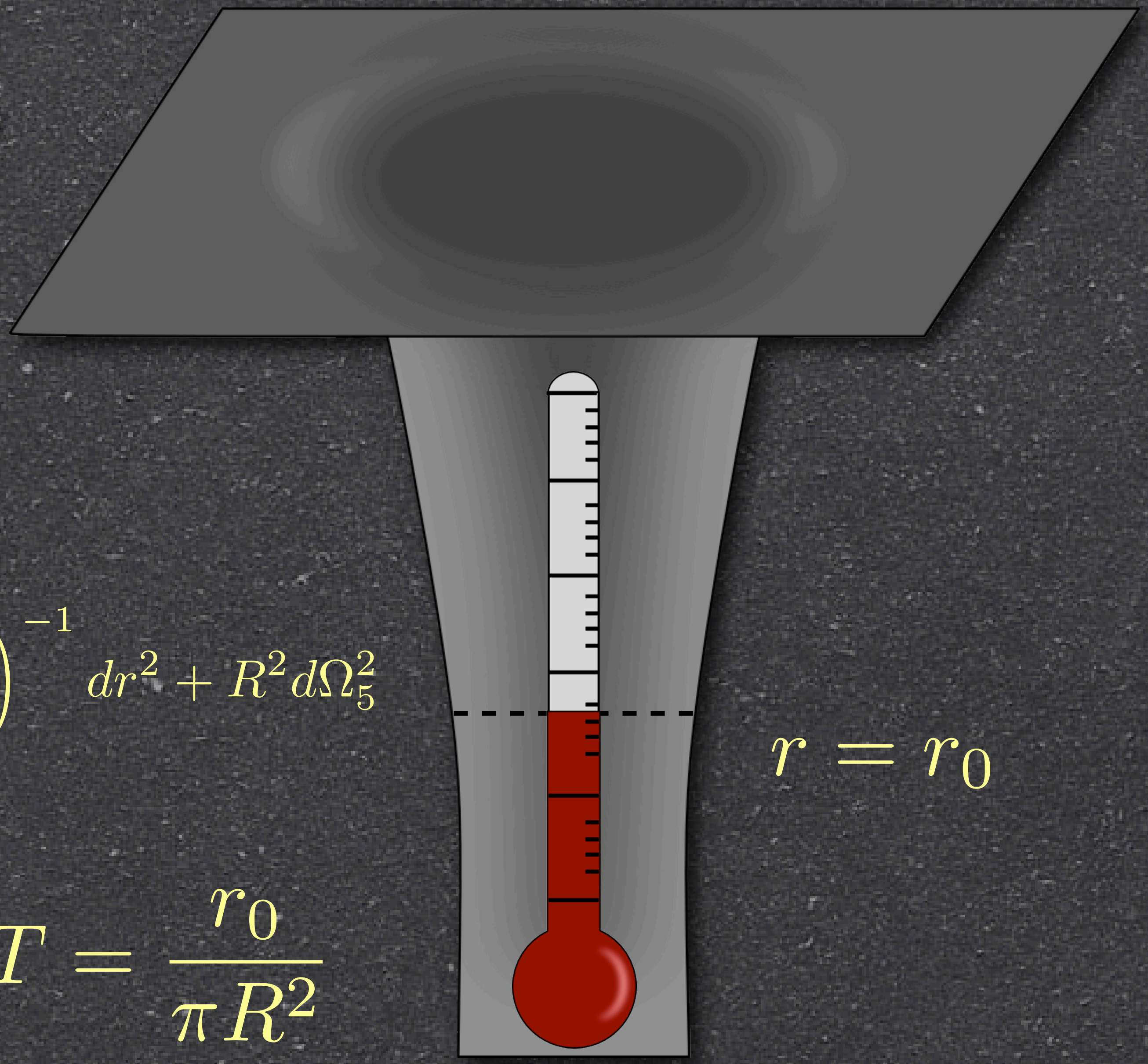
AdS & QCD

What gravity theory looks like hot QCD?

Need a temperature,
entropy,
hydrodynamics...

...like a black hole!

$$ds^2 = \frac{r^2}{R^2}\left[-\left(1-\frac{r_0^4}{r^4}\right)dt^2 + d\vec{x}^2\right] + \frac{R^2}{r^2}\left(1-\frac{r_0^4}{r^4}\right)^{-1} dr^2 + R^2 d\Omega_5^2$$

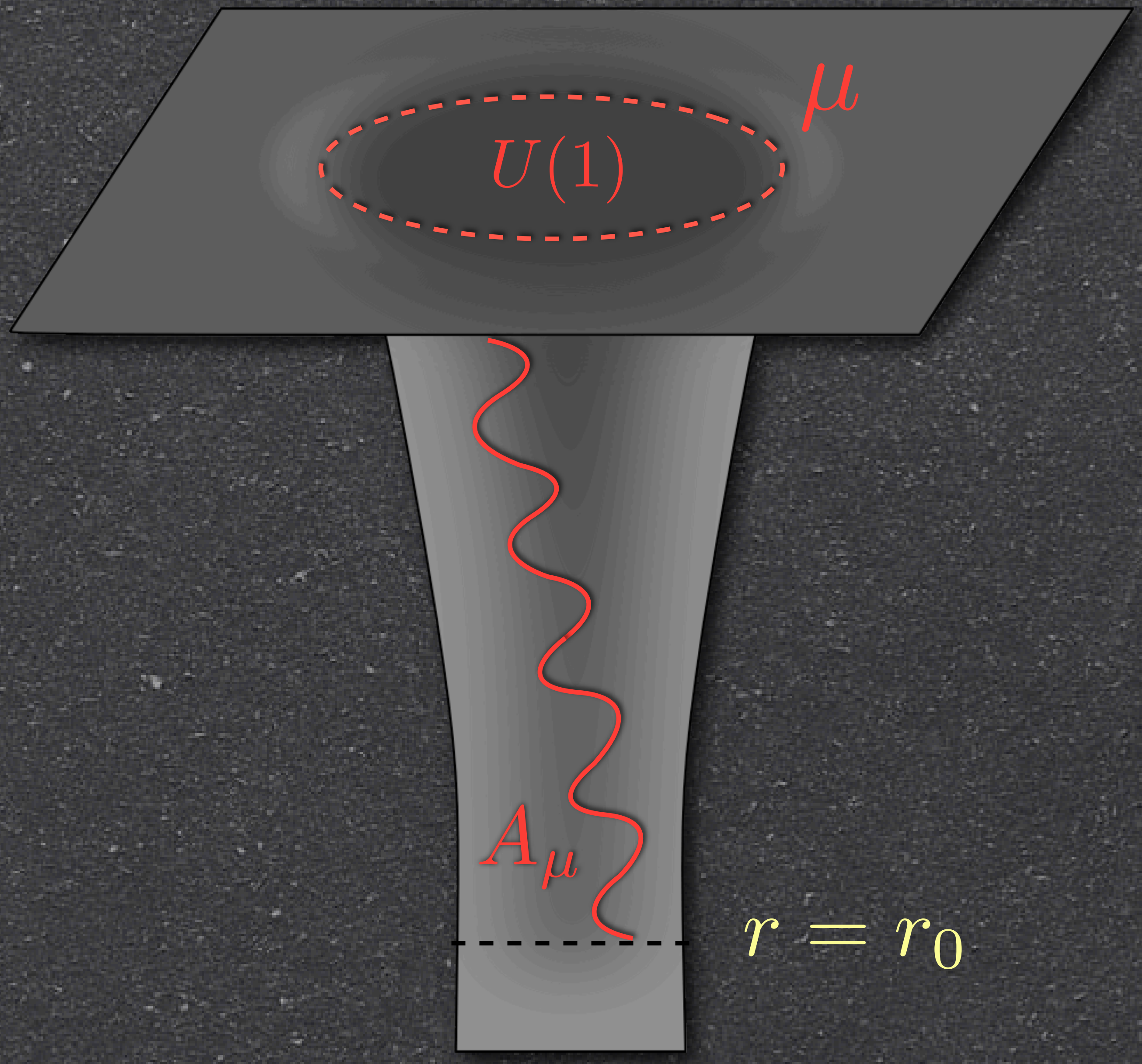


Hawking temperature: $T = \frac{r_0}{\pi R^2}$

AdS & QCD

What gravity theory looks like hot QCD?

Bulk gauge field ::
Boundary chemical potential



AdS & QCD

What gravity theory looks like hot QCD?

Bulk scalar field ::
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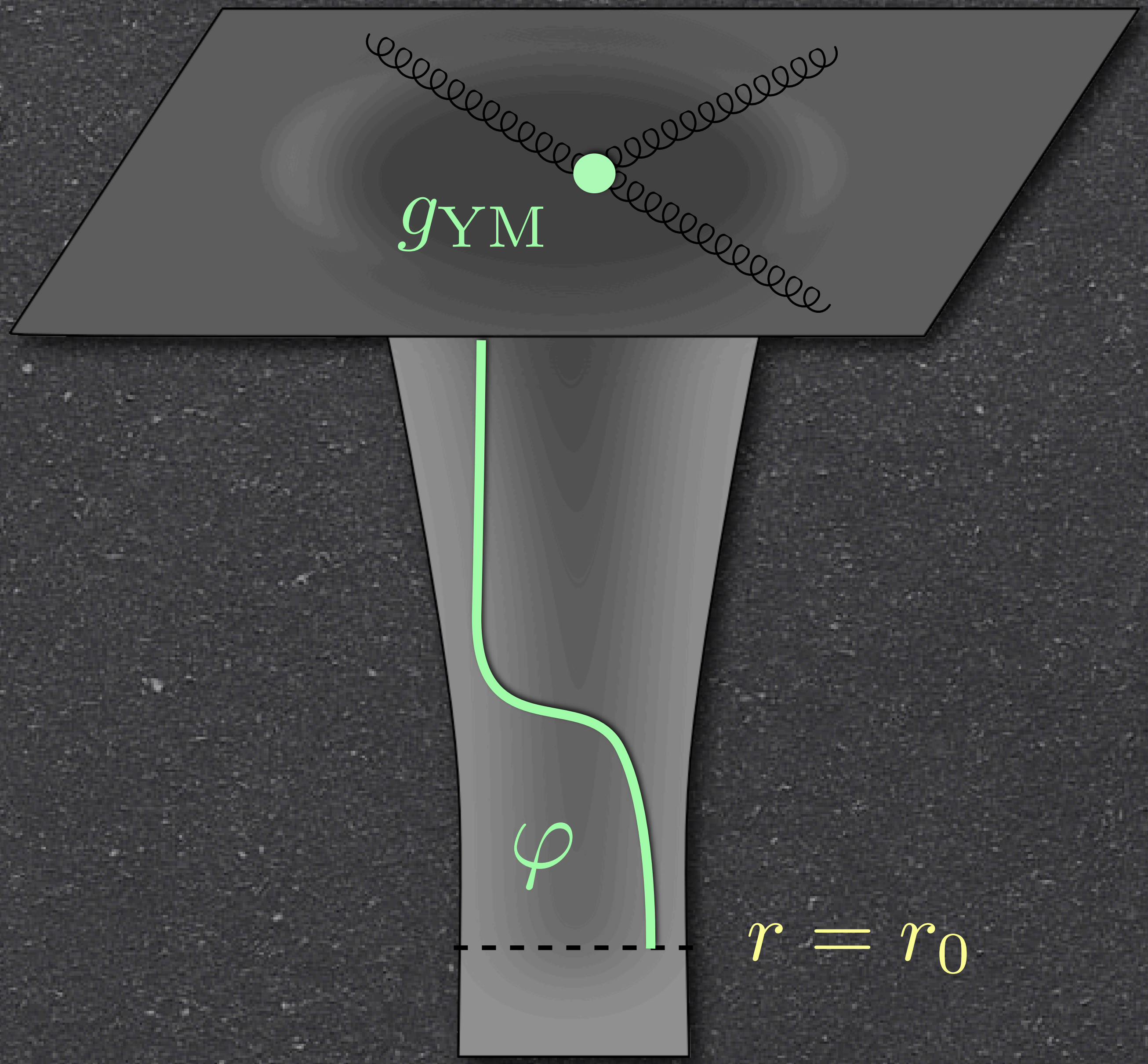


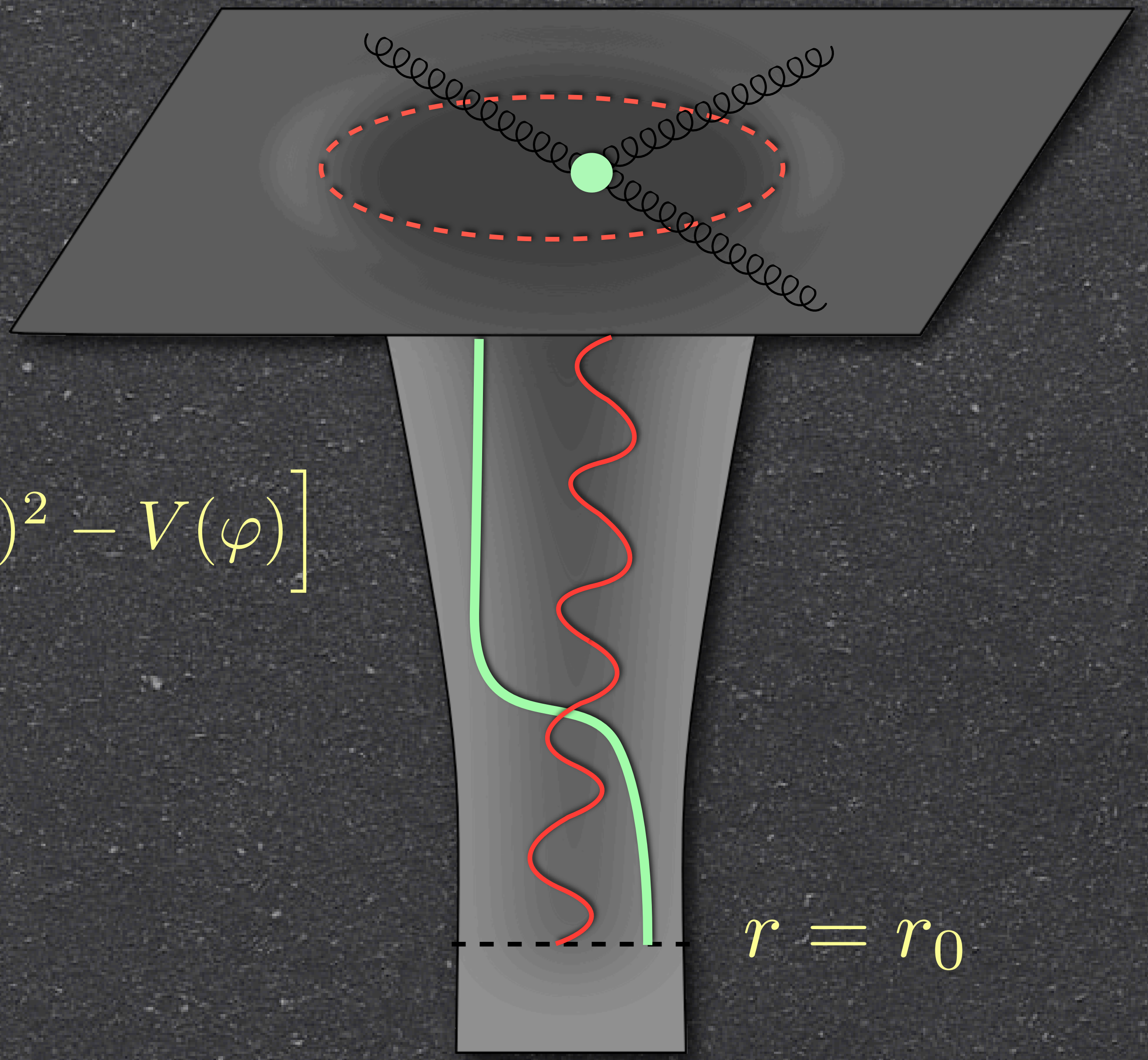
AdS & QCD

What gravity theory looks like hot QCD?

Bulk scalar and gauge field coupled to BH background:

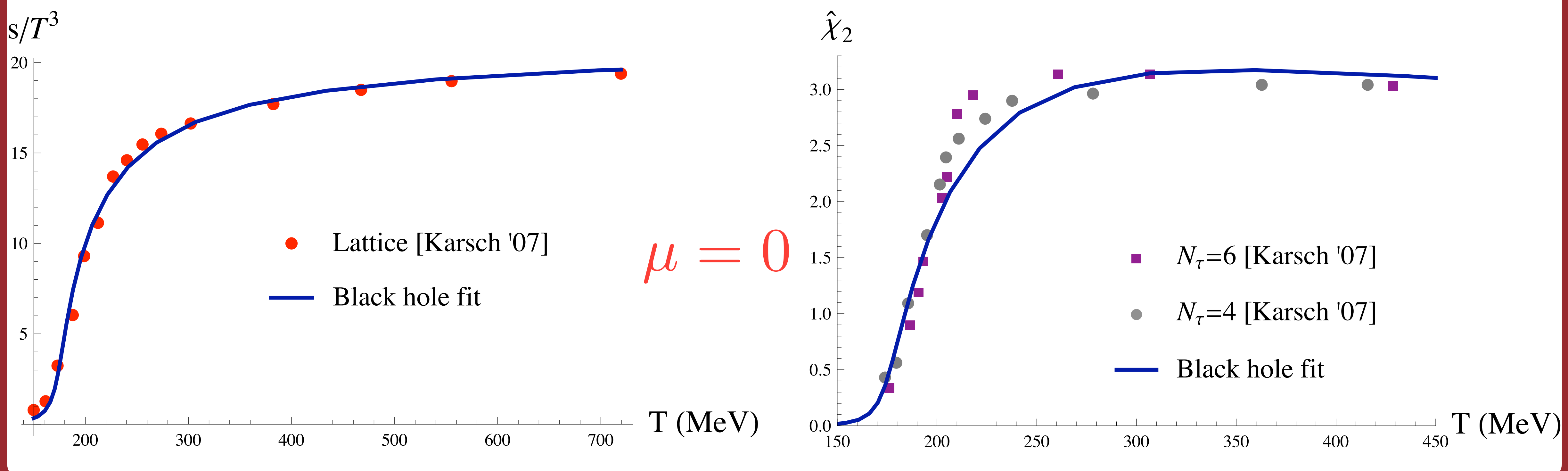
$$e^{-1}\mathcal{L} = \frac{1}{2\kappa^2}\left[R - \frac{f(\varphi)}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}(\partial\varphi)^2 - V(\varphi)\right]$$

Non-conformal gauge theory at finite temperature and chemical potential



AdS & QCD

What gravity theory looks like hot QCD?



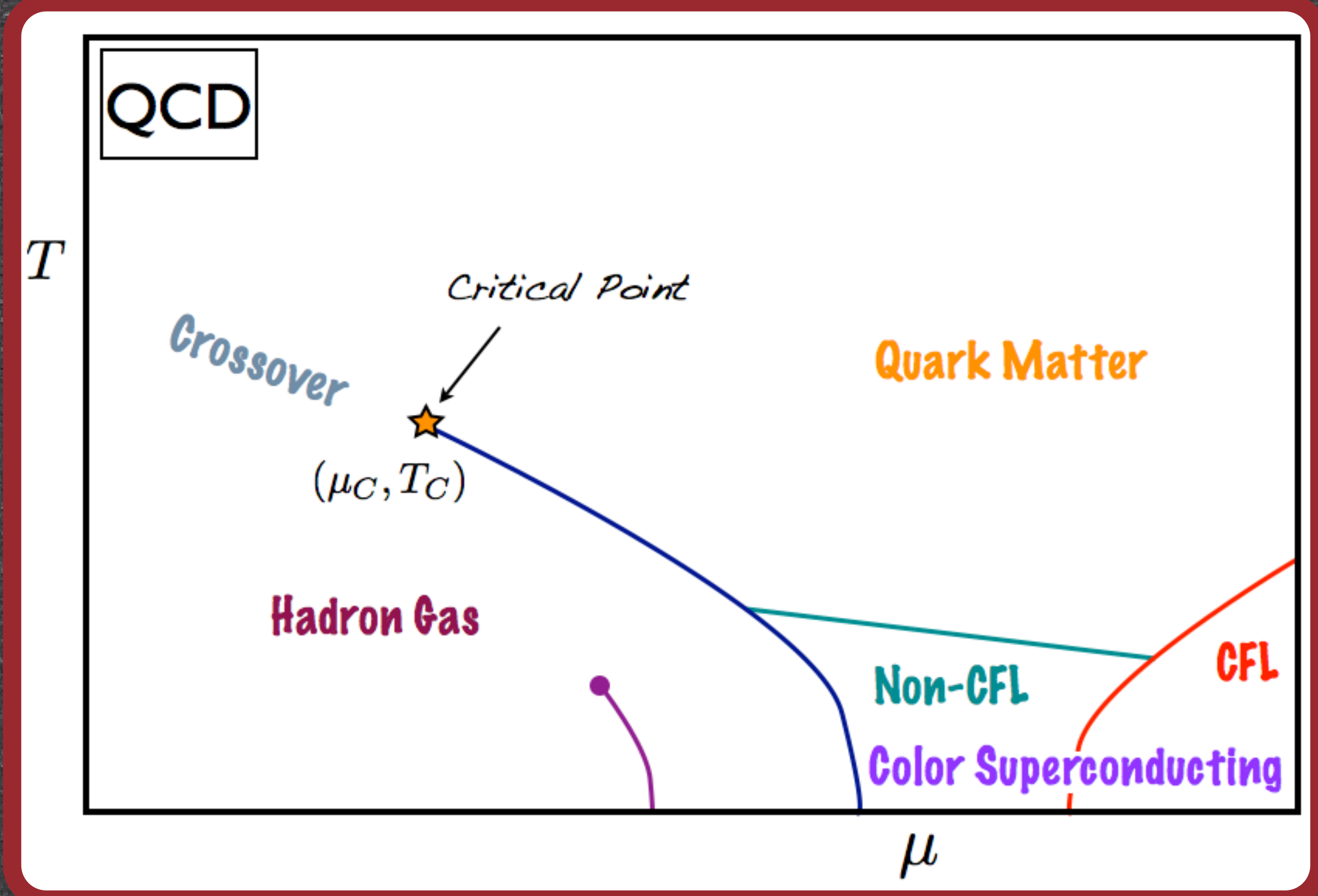
$$V(\varphi) = -12\cosh\gamma\varphi + b\varphi^2$$

$$f(\varphi) = \frac{\operatorname{sech}\left[\frac{6}{5}(\varphi - 2)\right]}{\operatorname{sech}\frac{12}{5}}$$

AdS & QCD

What can this model teach us about QCD?

Our system is easy to study at finite temperature and chemical potential!

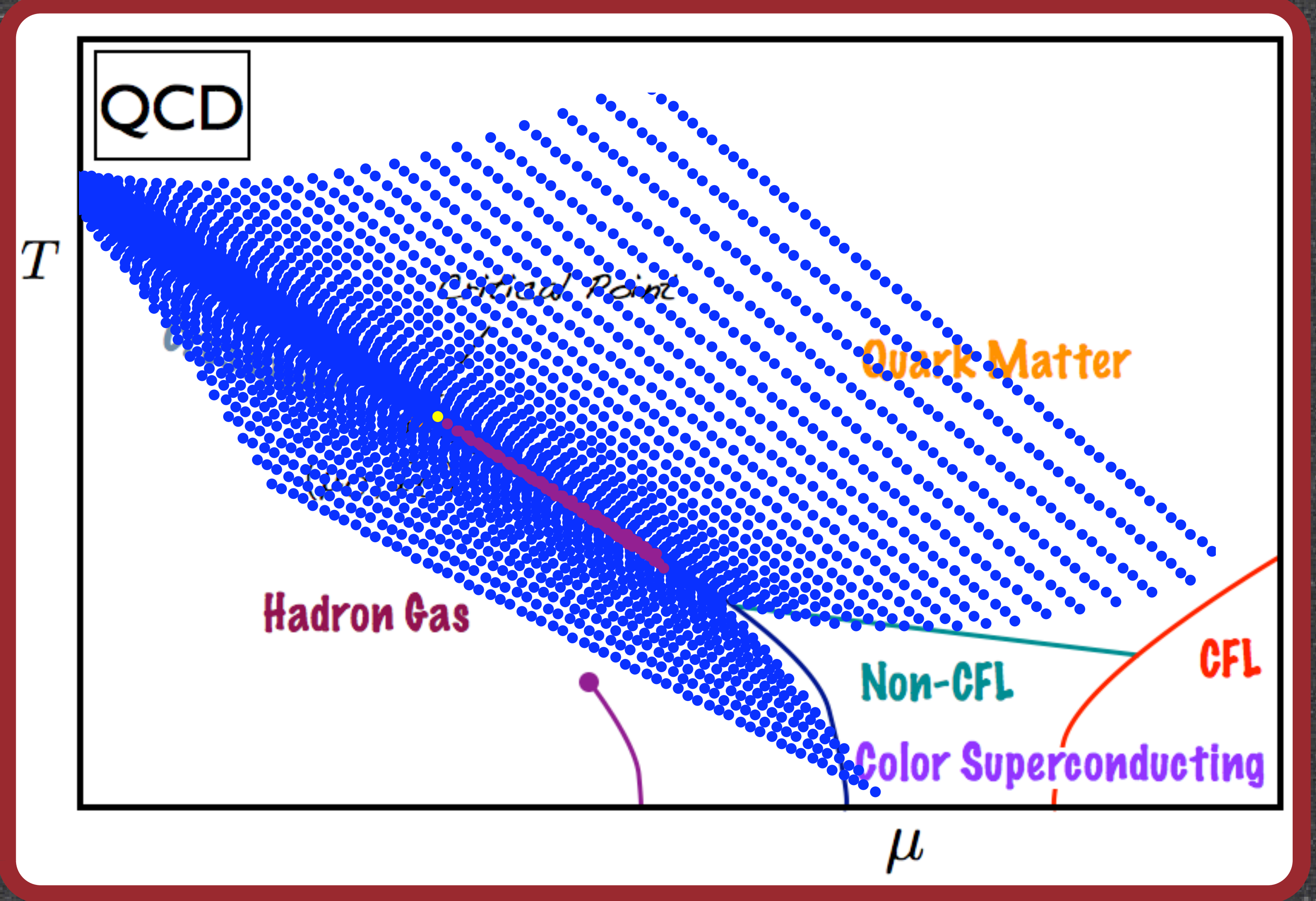


AdS & QCD

What can this model teach us about QCD?

Approach: populate the mu-T plane with bulk solutions and search for a line of thermodynamic instabilities...

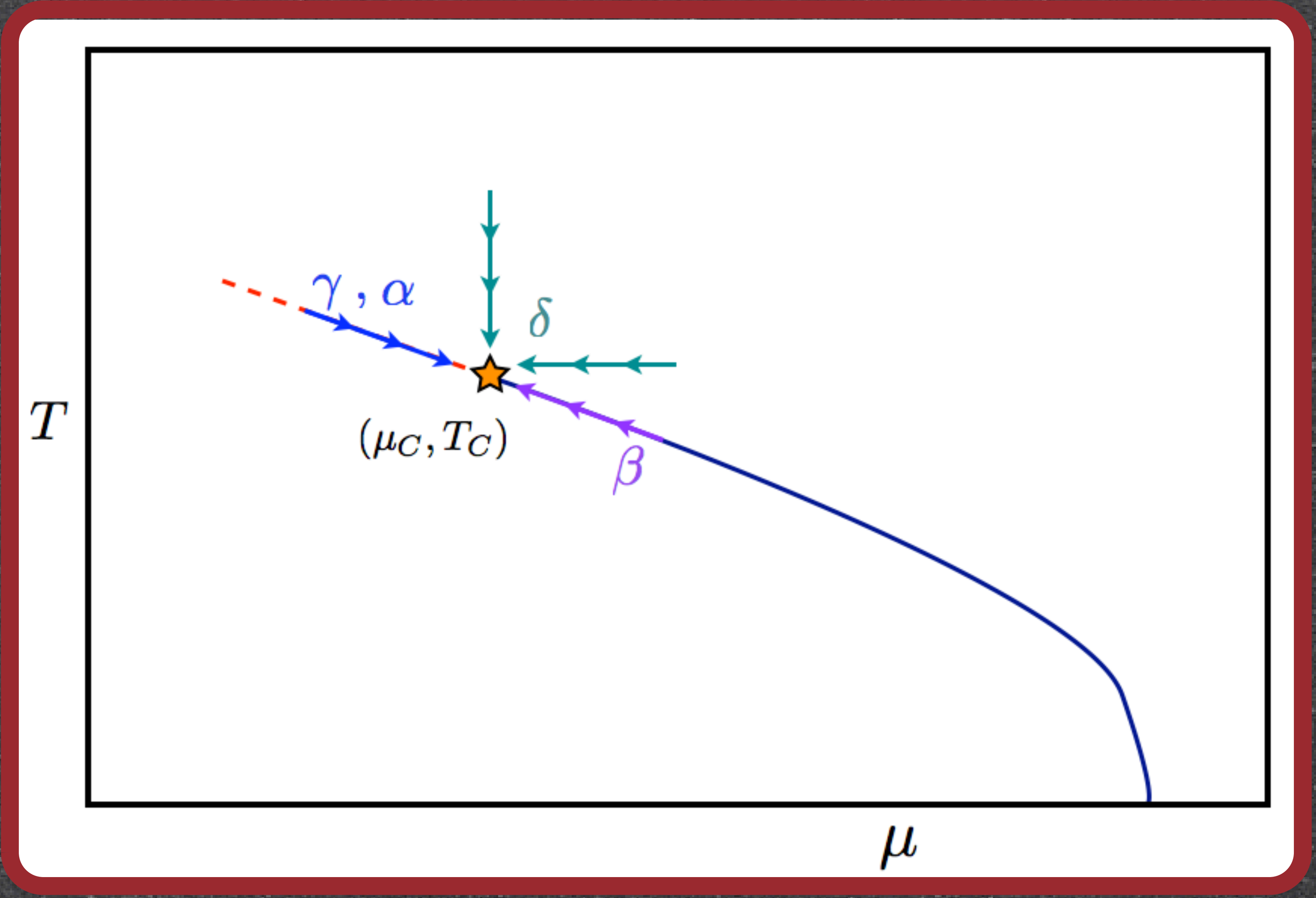
...locate critical point!



$$(\mu_c, T_c) = (783\ \mathrm{MeV}, 143\ \mathrm{MeV})$$

AdS & QCD

What can this model teach us about QCD?



Critical points are categorized by critical exponents:

$$\Delta\rho \sim (T_c - T)^{\beta}$$

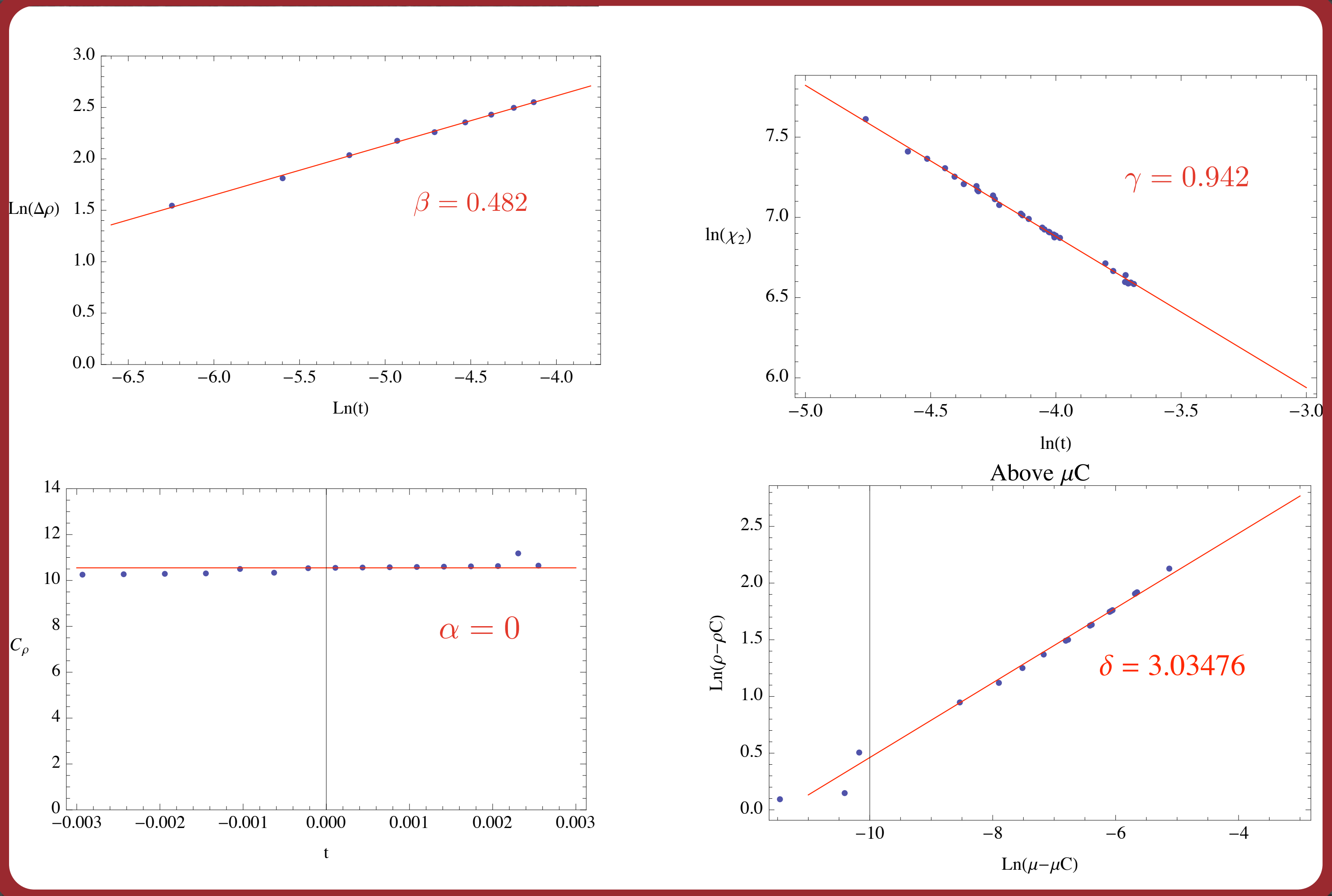
$$\rho - \rho_c \sim |\mu - \mu_c|^{1/\delta}$$

$$C_\rho \sim |T - T_c|^{-\alpha}$$

$$\chi_2 \sim |T - T_c|^{-\gamma}$$

AdS & QCD

What can this model teach us about QCD?



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AdS & QCD

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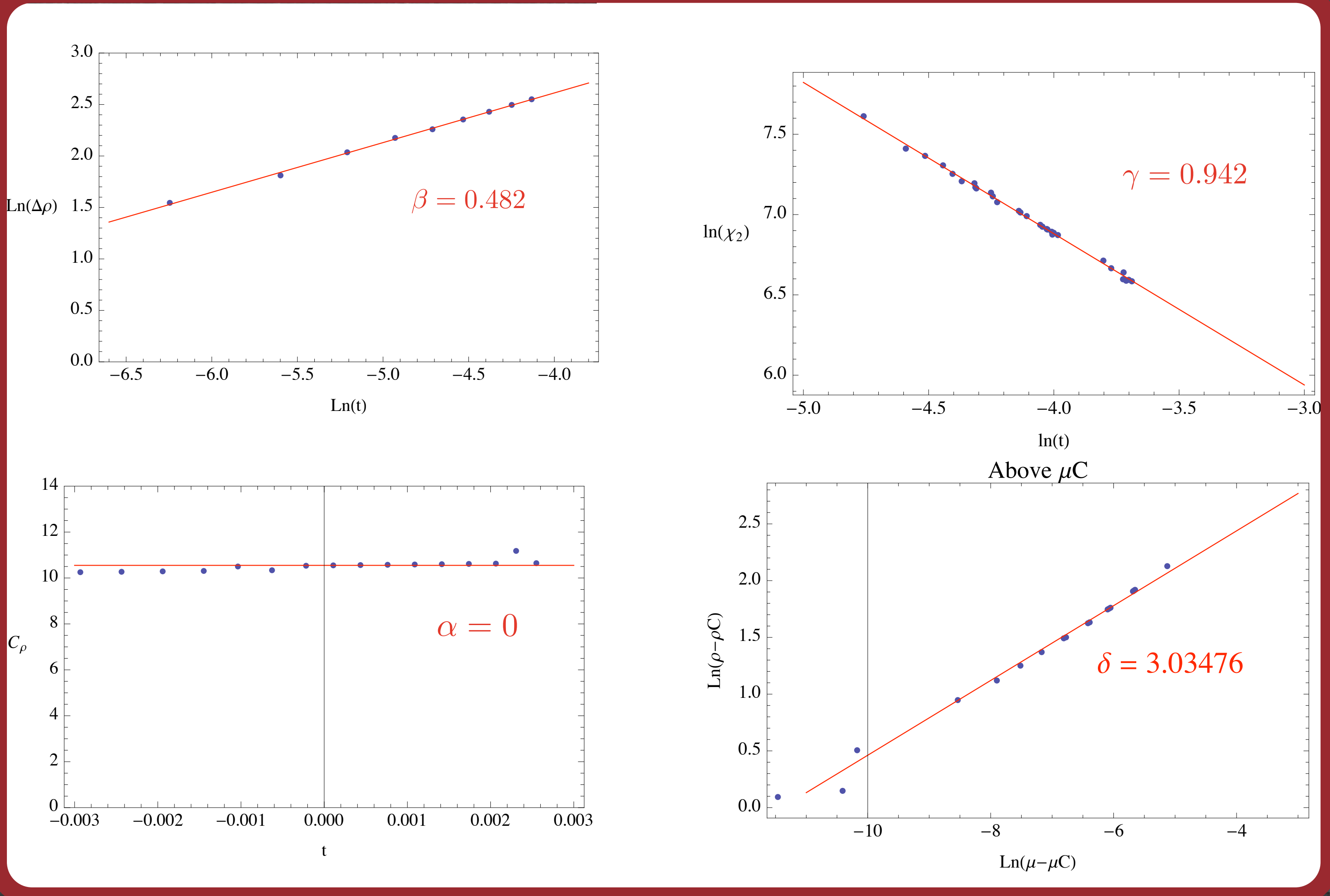

This suggests mean field 3d Ising!

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AdS & QCD

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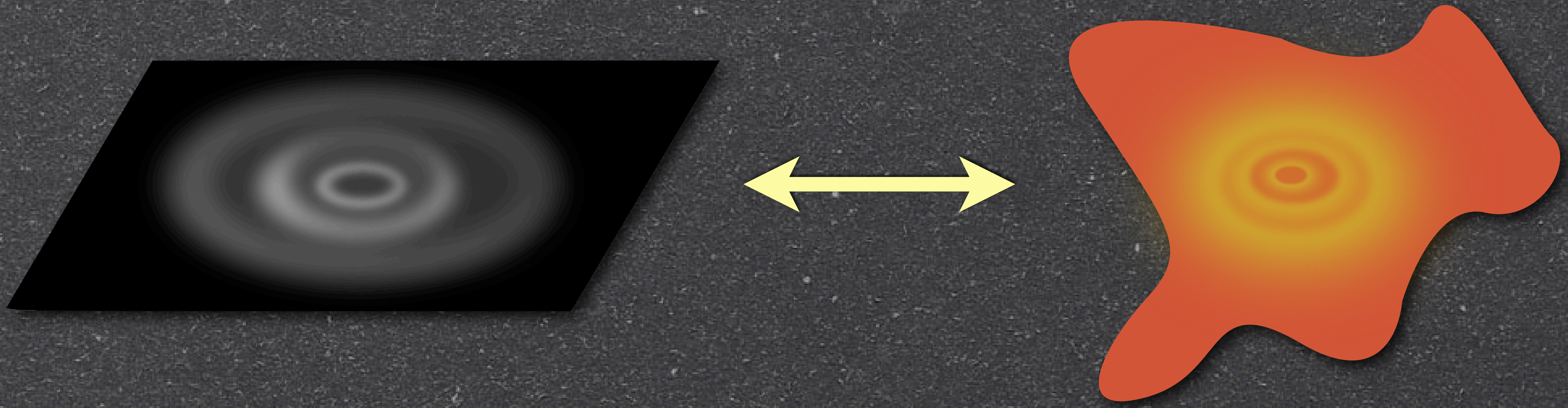
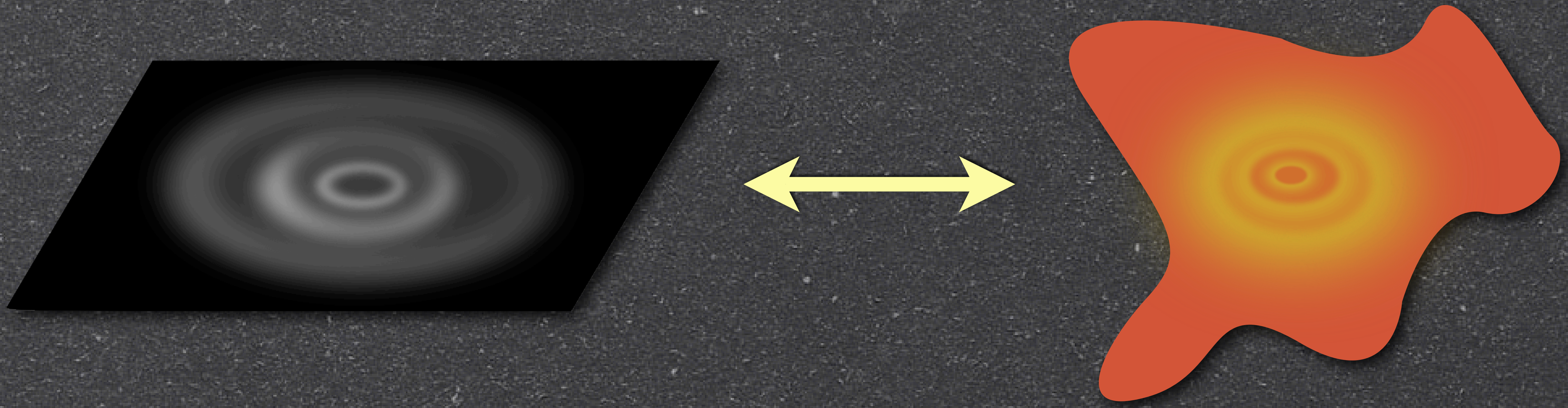


Fluctuations of the bulk theory map to hydrodynamic transport in the boundary theory.

AdS & QCD

What can this model teach us about QCD?

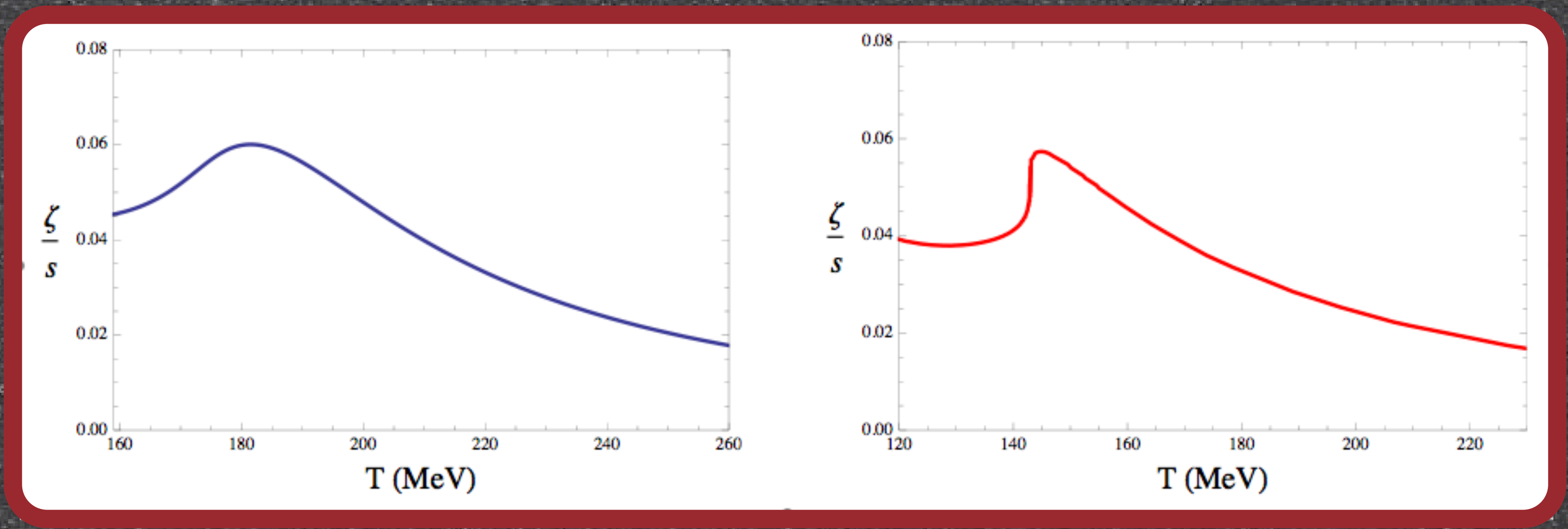


Viscosities, conductivities, and diffusion are calculable via standard techniques, e.g. Kubo:

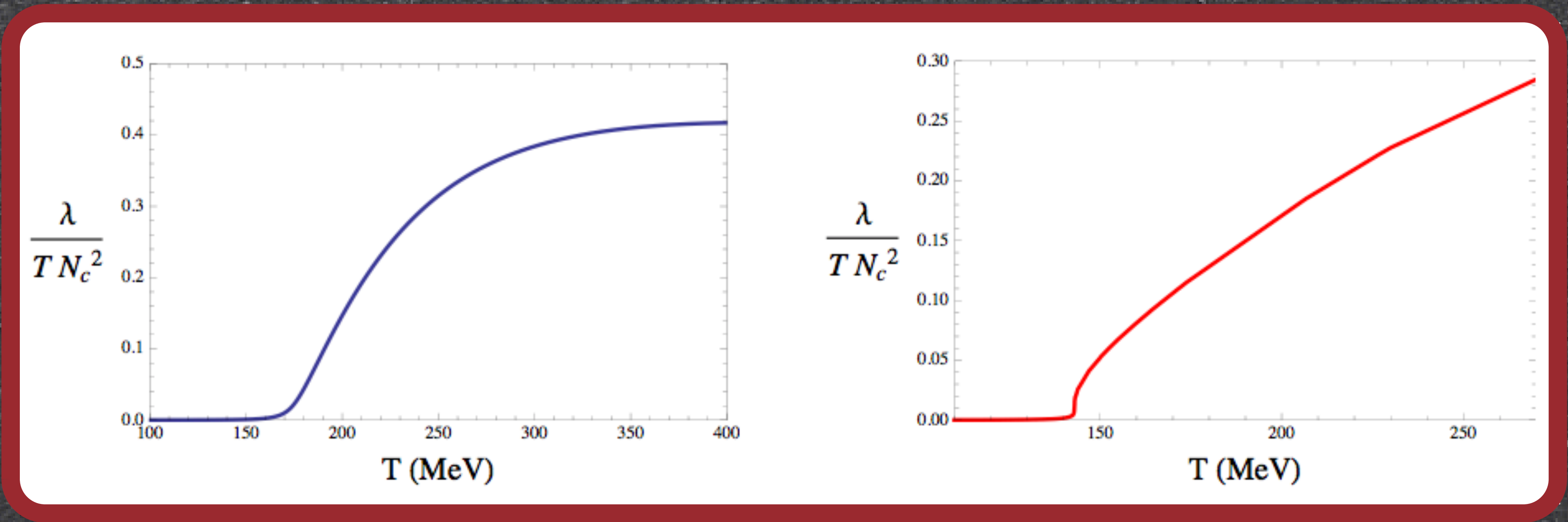
$$h_{ij} \rightsquigarrow T_{ij} \qquad \text{and} \qquad \zeta \sim \lim_{\omega \to 0} \frac{1}{\omega} \mathrm{Im}\, G_R{}^{i}{}_{i}{}^{k}{}_{k}(\omega)$$

AdS & QCD

What can this model teach us about QCD?



Bulk viscosity and conductivity are finite at critical point



Hohenberg & Halperin Model H --> Model B via large N?

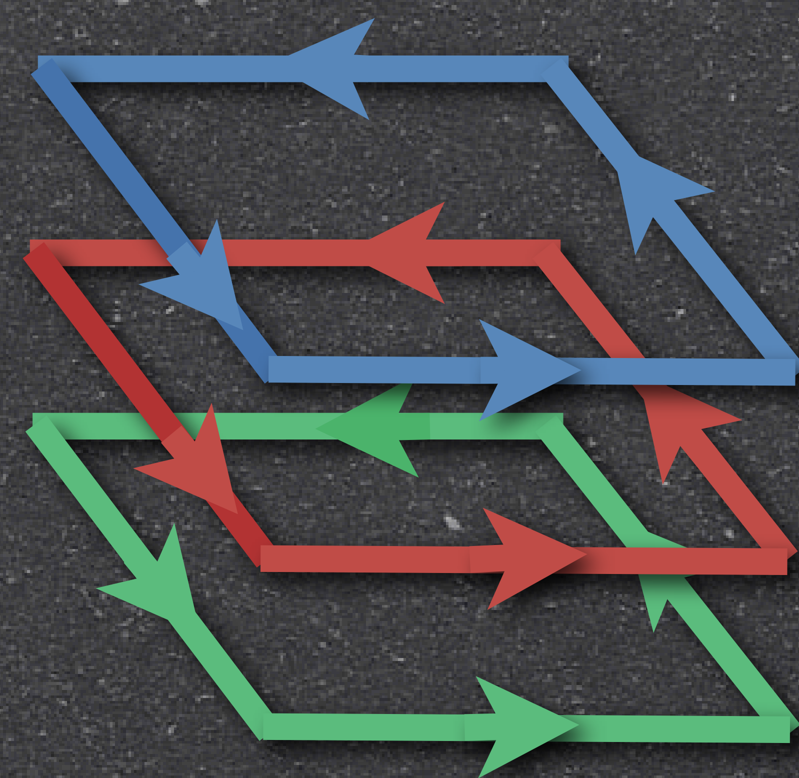
AdS & QCD

What can this model teach us about lattice QCD?

Finite chemical potential on lattice

↕

“sign problem”

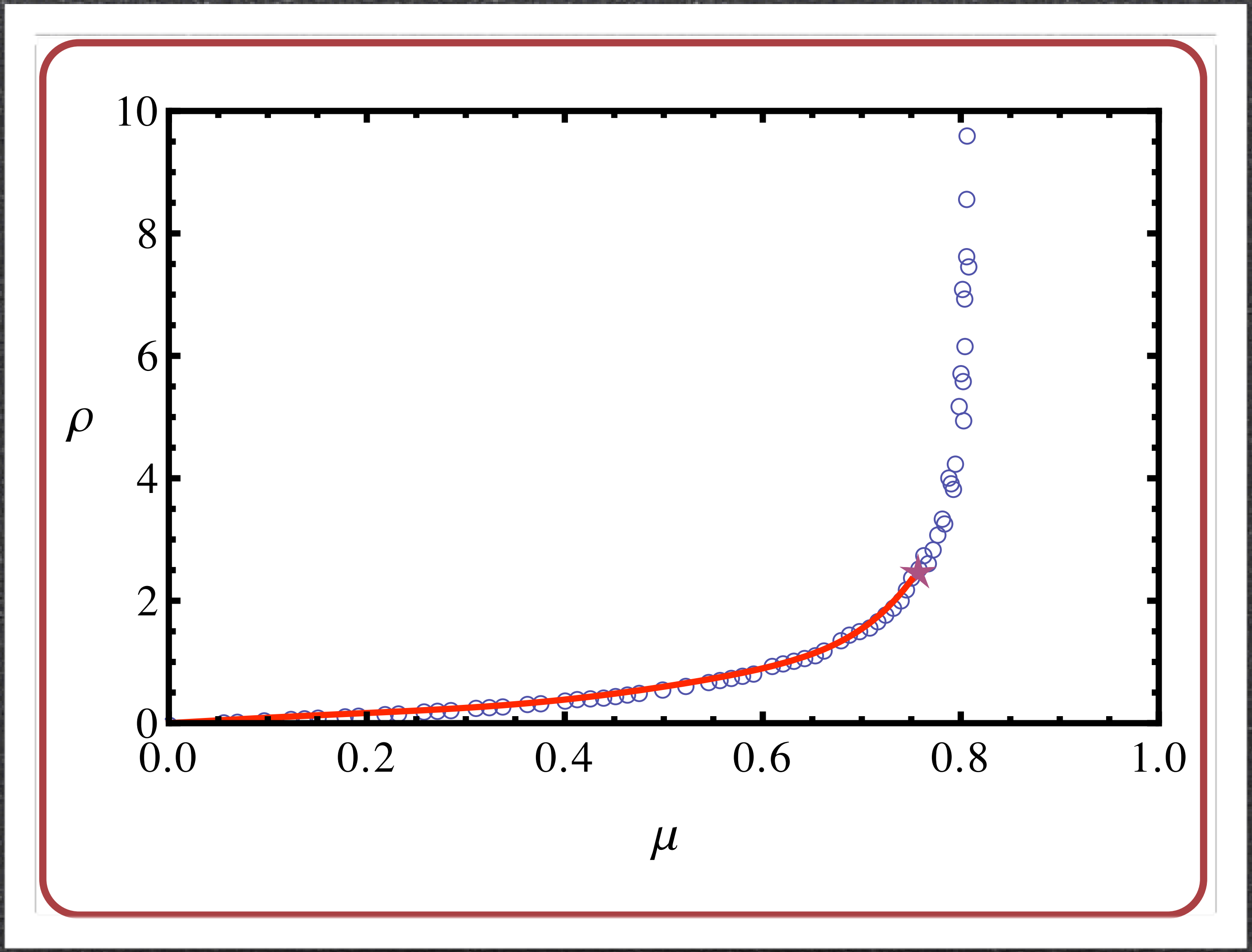


To circumvent, try Taylor:

$$\frac{P(T,\mu)}{T^4} = \sum_n c_{2n}(T) \left(\frac{\mu}{T}\right)^{2n}$$

AdS & QCD

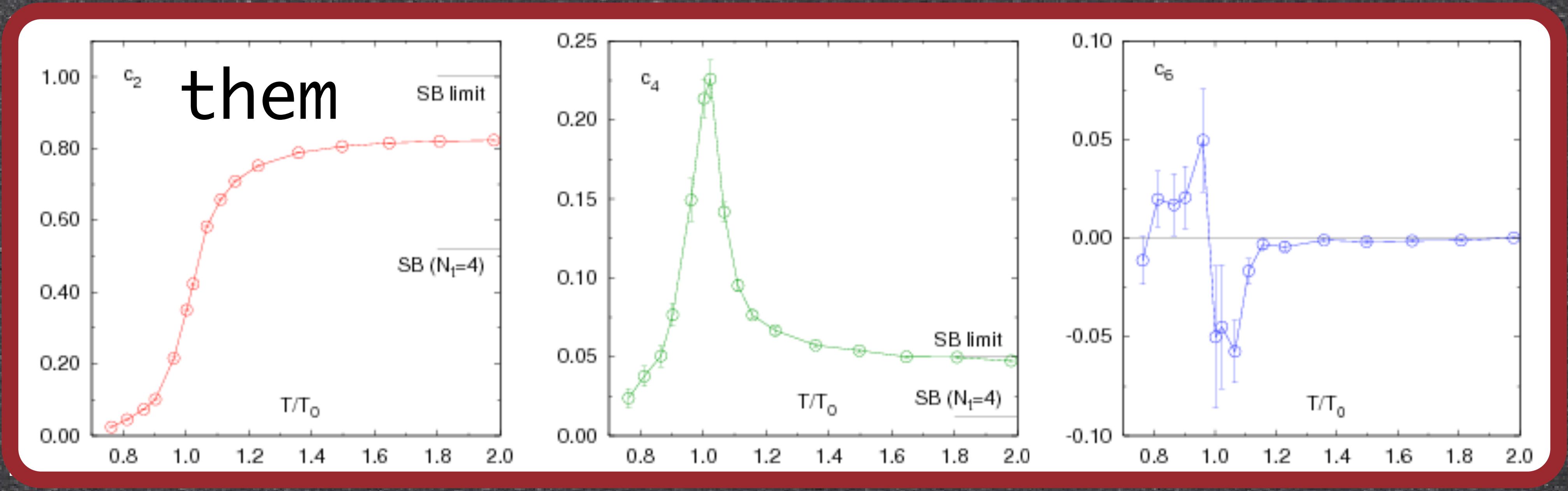
What can this model teach us about lattice QCD?

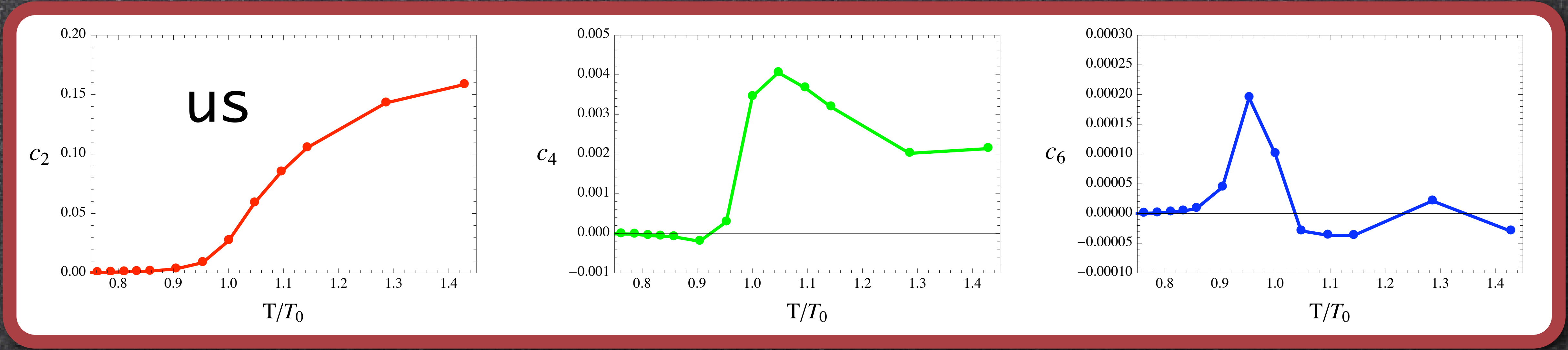


Approach: Fit our data to polynomial, extract power series coefficients, compare.

AdS & QCD

What can this model teach us about lattice QCD?





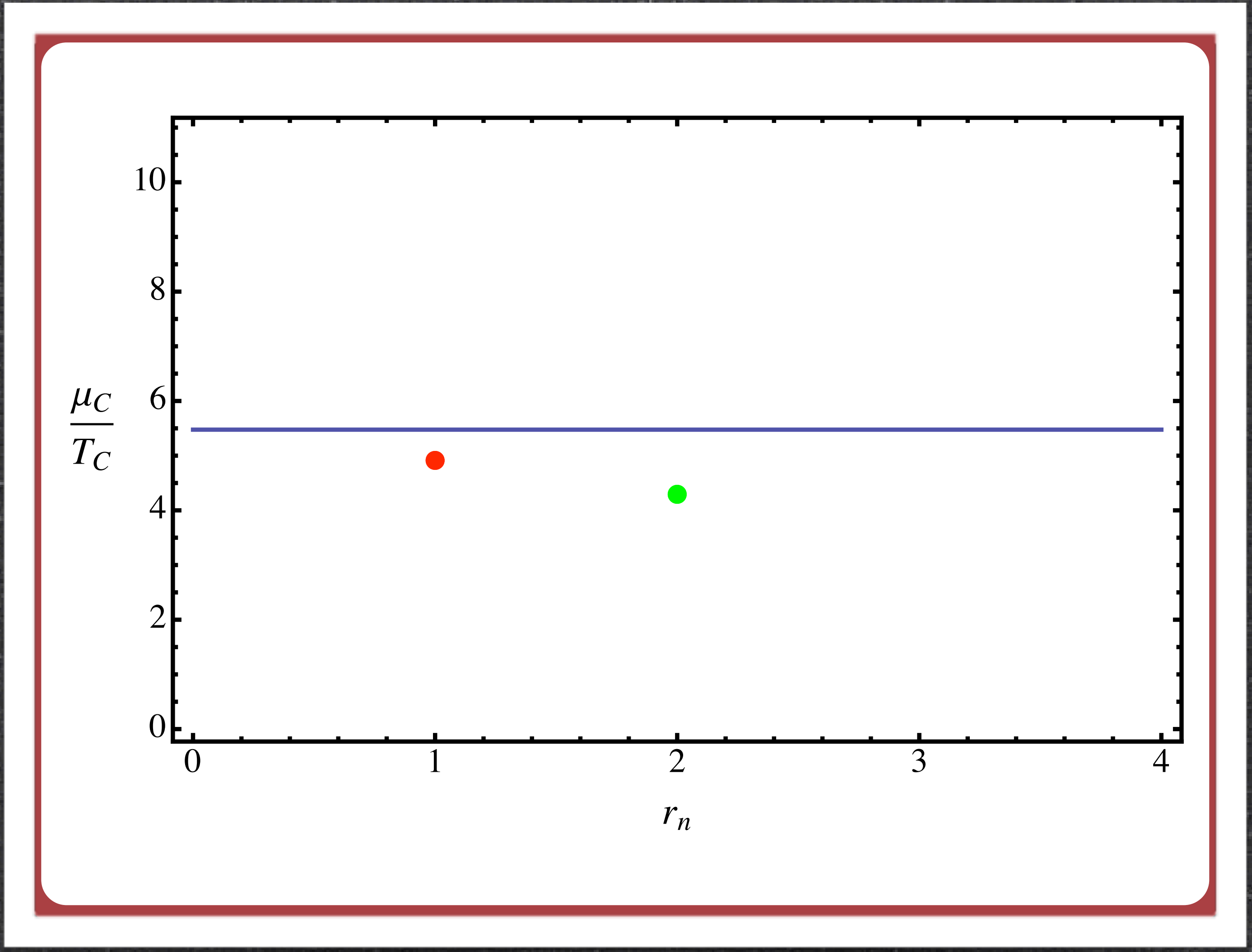
AdS & QCD

What can this model teach us about lattice QCD?

$$\frac{\mu_C}{T_C} = \lim_{n\to\infty} \sqrt{\left|\frac{c_{2n}(T_C)}{c_{2n+2}(T_C)}\right|}$$

so...

$$\frac{\mu_C}{T_C} = 5.48 \approx 4.88, 4.26, \ldots ?$$



AdS & QCD

What can this model teach us about QCD?

- A calculable model of a critical gauge theory at finite mu, T: 1012.1864

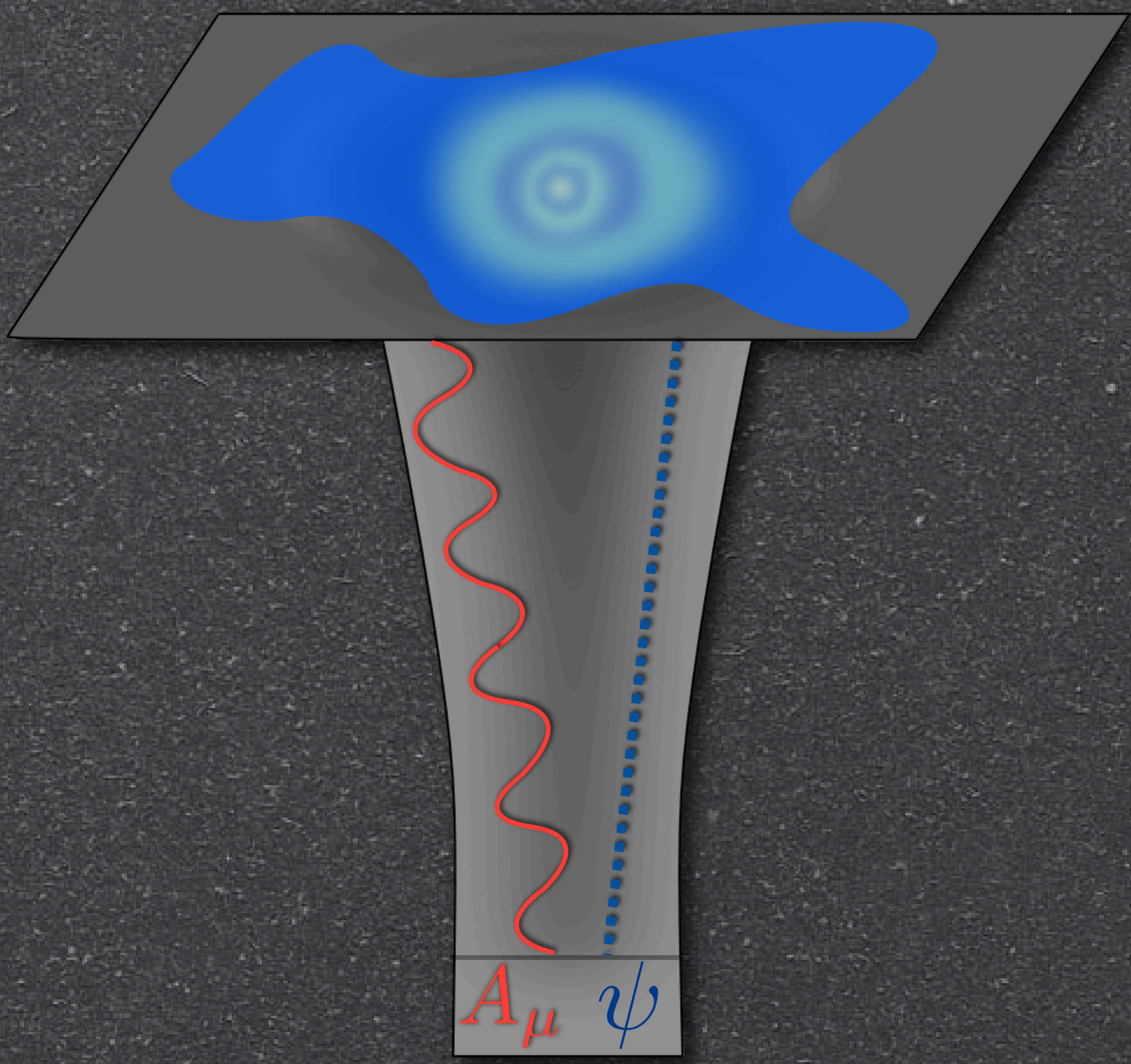
- Access to dynamic critical phenomena: 1108.2029

- Provides proving ground for lattice techniques: (in progress)

- Study sensitivity of results to bulk theory specifics: 0903.1458

AdS & CMT

What gravity theory looks like a non-Fermi liquid?



Fermi Surface:

Surface in momentum space where the zero frequency fermion correlator diverges.

↕

Fermion zero modes in extremal AdSRN!

AdS & CMT

Does such a model exist in string theory?

$$\mathcal{L}_{\rm SUGRA} = \mathcal{L}[e, A, \lambda, \psi, \phi, \ldots] \to \mathcal{L}[e, A, \psi]$$

AdS & CMT

Does such a model exist in string theory?

$$\mathcal{L}_{\text{SUGRA}} = \mathcal{L}[e, A, \lambda, \psi, \phi, \ldots] \to \mathcal{L}[e, A, \psi]$$

Evidently--in maximal gauged SUGRA theories:

D = 4	D = 5
$\mathcal{N} = 8$ $SO(8)$ SUGRA on AdS_4	$\mathcal{N} = 8$ $SO(6)$ SUGRA on AdS_5
↕	↕
d=3 $\mathcal{N} = 8$ ABJM theory	d=4 $\mathcal{N} = 4$ SYM theory

AdS & CMT

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Evidently--in maximal gauged SUGRA theories:

D = 4

$$\gamma^\mu \left(\nabla_\mu - i \frac{1}{\sqrt{2}L} a_\mu \right) \psi = 0$$

D = 5

$$\left(\gamma^\mu \nabla_\mu - i \frac{5}{L} \gamma^\mu a_\mu - \frac{1}{2L} - \frac{i}{4} f_{\mu\nu} \gamma^{\mu\nu} \right) \psi = 0$$

This is a “top down” example of AdS/CMT!

AdS & CMT

What can this model teach us about CMT?

- Embedding of Fermi surface with basic study of near surface excitations: 1112.3036
- Other applications to, e.g cold atomic systems. By modifying spacetime asymptotics, develop duals to charged, rotating non-relativistic CFTS: 0907.1920

The Longview

- Understanding the sensitivity of our toy QCD model's predictions to Lagrangian parameters is important.
- Further exploration of lattice techniques in gauge/gravity models is an exciting avenue.
- Many details related to the interpretation of Fermi surface embeddings remain.