2.3 Galactic Rotation

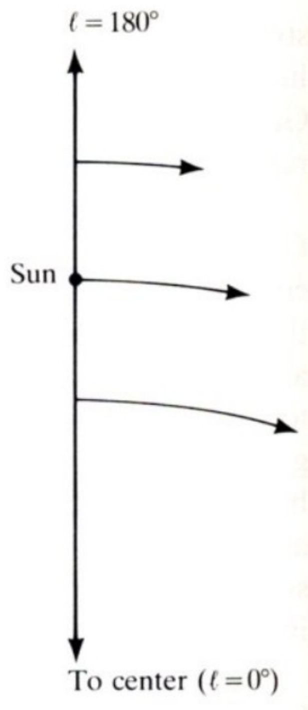
OVERVIEW

- Local rotation and Oort constants
- Rotation curve

1 – LOCAL ROTATION

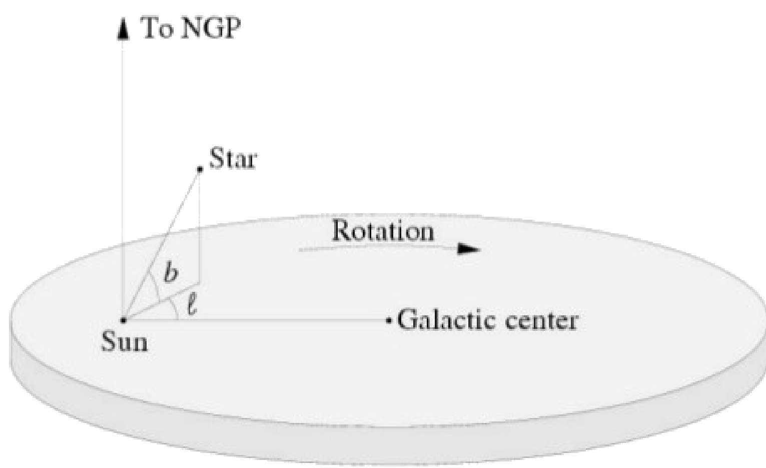
- We will only consider the *local* rotation

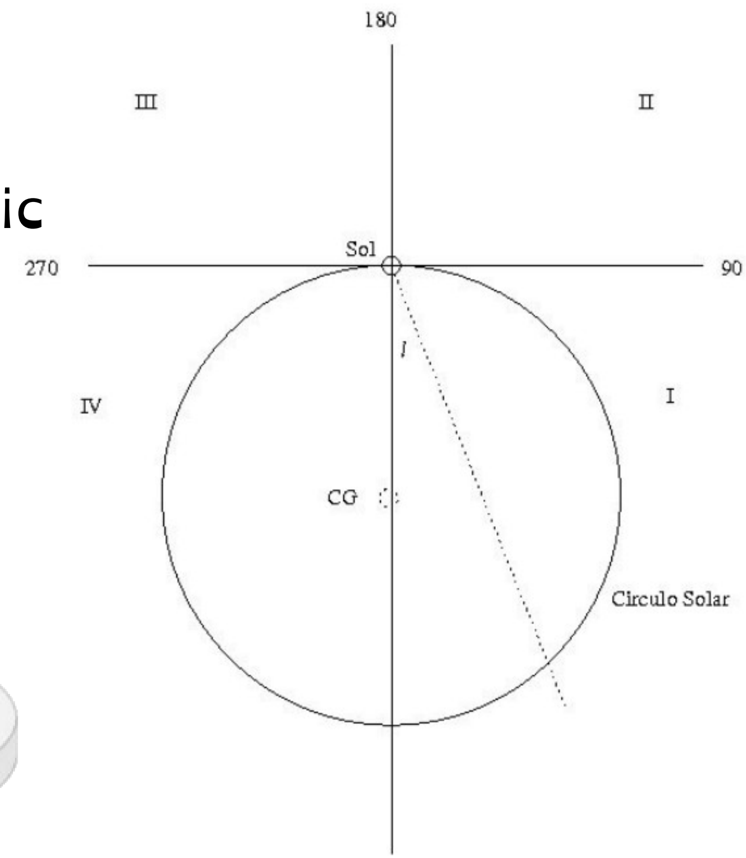
  Around $R_0 \sim 8$ kp the rotation curve is slightly "descending"



- Differential rotation

  → objects closer to Galactic center rotate faster

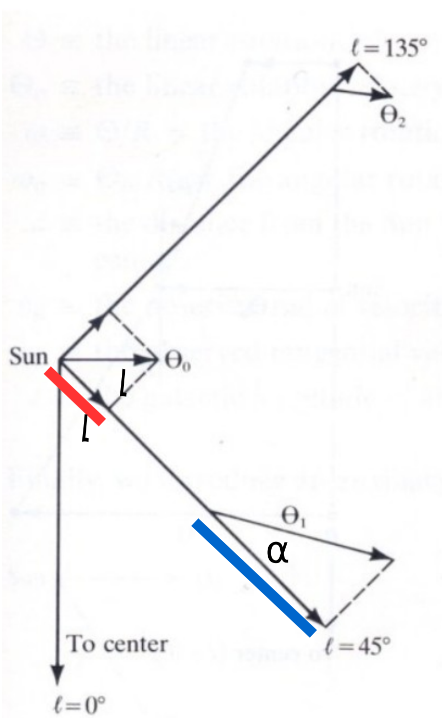




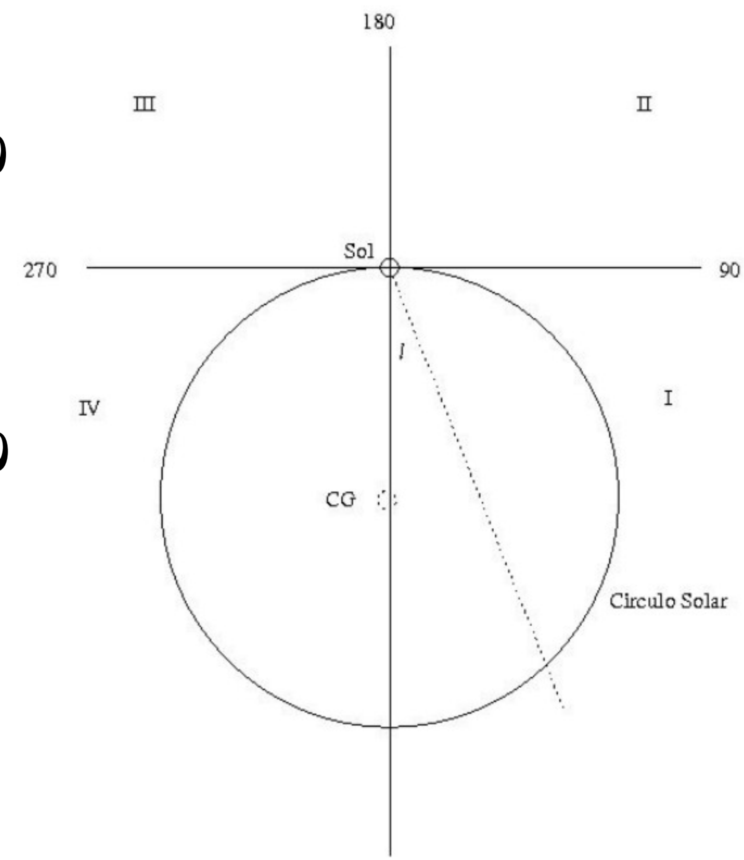
LINE OF SIGHT VELOCITY

- Velocity along to line of sight (v_{LOS}), seen from Sun:

$$\Theta\cos(\alpha) - \Theta_0\sin(l)$$

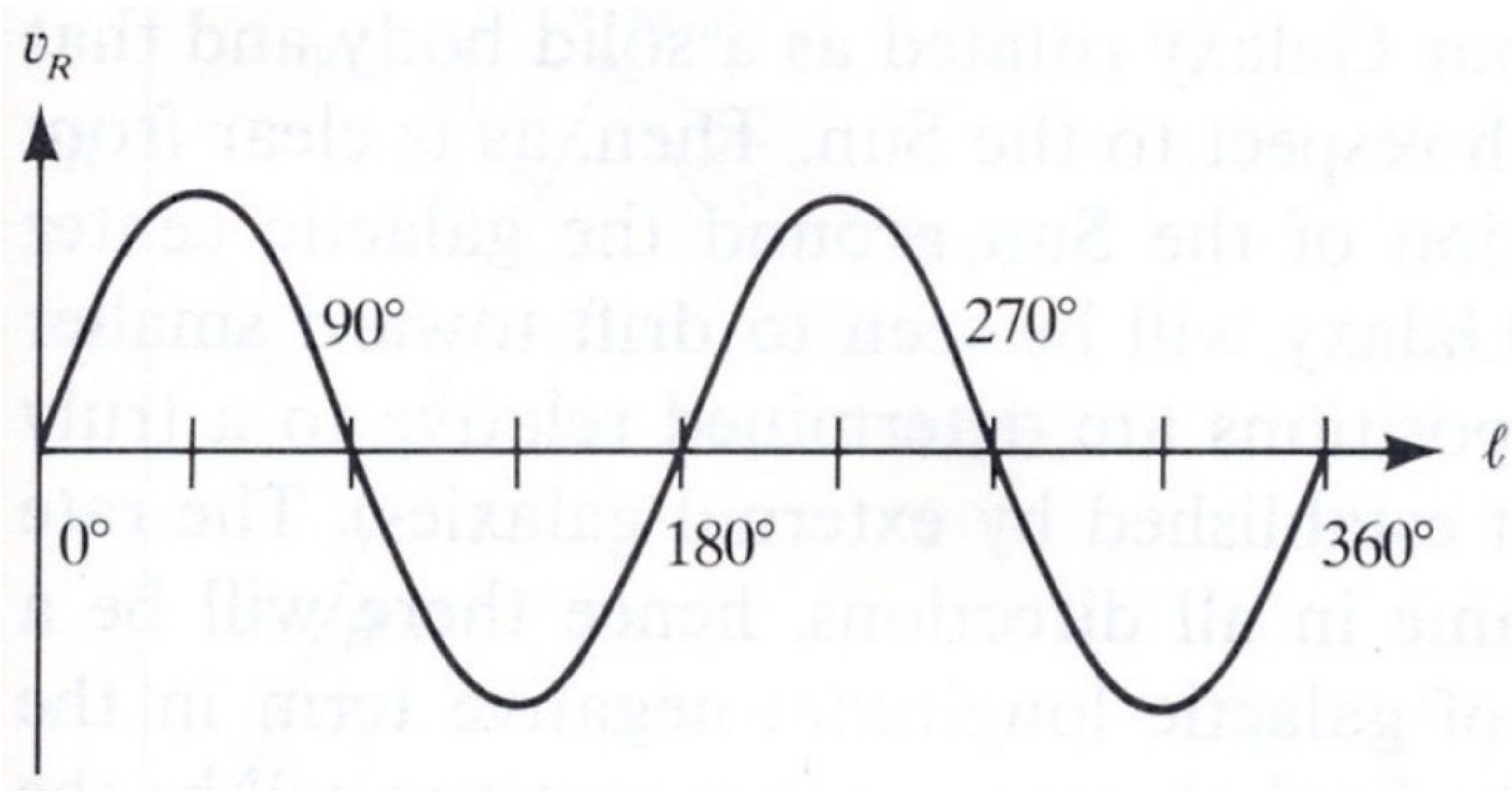


- I quadrant → $v_{LOS} > 0$
- II quadrant → $v_{LOS} < 0$
- III quadrant → $v_{LOS} > 0$
- IV quadrant → $v_{LOS} < 0$



LINE OF SIGHT VELOCITY

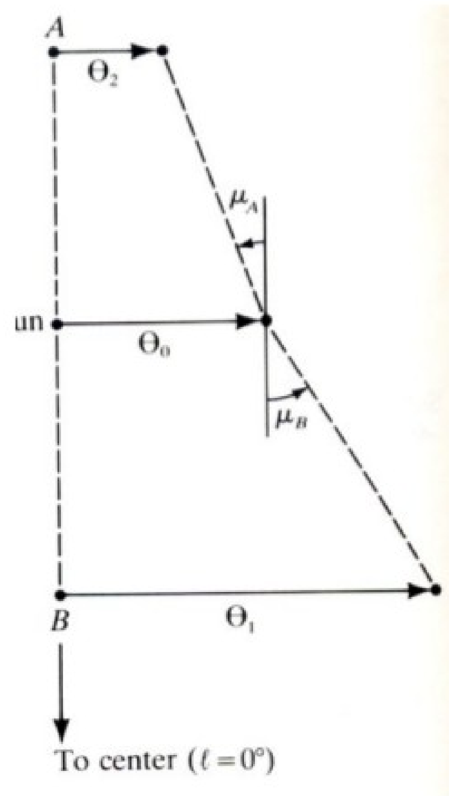
- In general, v_{LOS} goes roughly as *sin(2l)*:



TANGENTIAL VELOCITY

- Similarly, we can show that v_{POS} (tangential velocity) ≥ 0

  (Independently of quadrant)



LINE OF SIGHT VELOCITY - EXACT FORMULATION

- We will now on assume:
  - the Galactic disk is infinitesimally thin
  - stars move on circular orbits

- In Galactic coordinates, but from p.o.v. of the Sun:

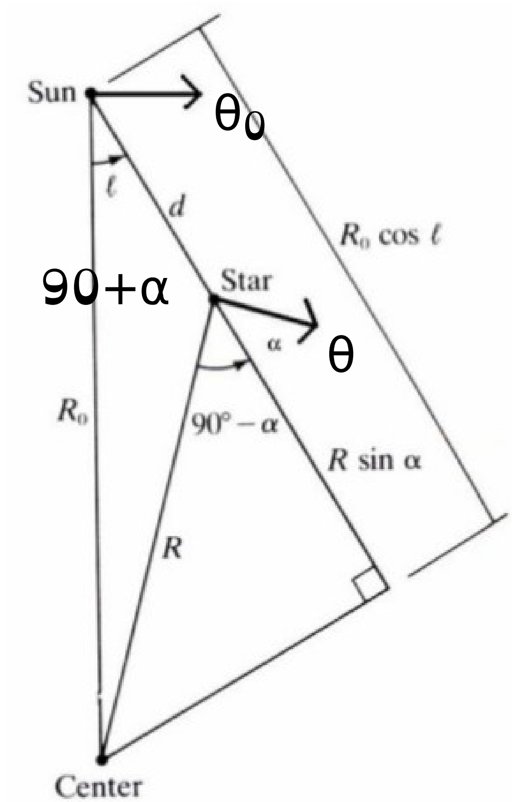
$$v_{LOS} = \Theta \cos\alpha - \Theta_0 \sin l$$

$$\frac{\sin l}{R} = \frac{\sin(\alpha + \pi/2)}{R_0} = \frac{\cos\alpha}{R_0}$$

$$v_{LOS} = \left(\frac{\Theta R_0}{R}\right)\sin l - \Theta_0 \sin l$$

$$\omega = \Theta/R$$

$$v_{LOS} = (\omega - \omega_0) R_0 \sin l$$



TANGENTIAL VELOCITY - EXACT FORMULATION

- Tangential velocity:

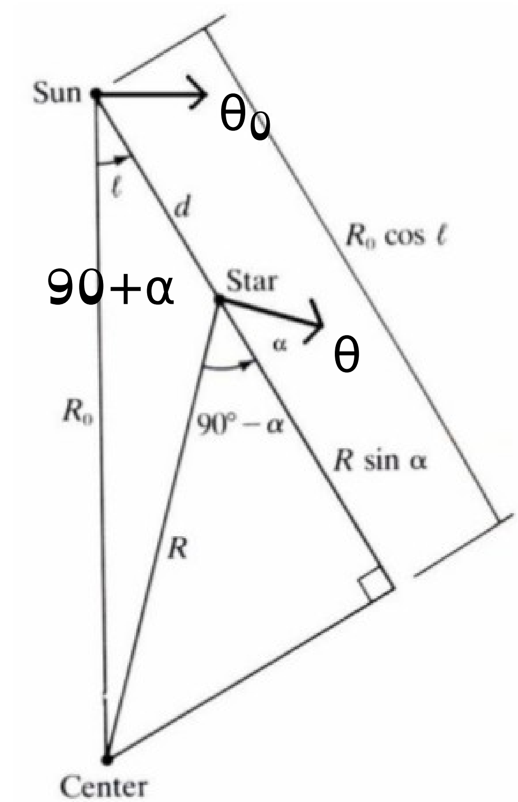
$$v_{POS} = \Theta \sin\alpha - \Theta_0 \cos l$$

$$R_0 \cos l = R \sin\alpha + d$$

$$v_{POS} = \Theta\left(\frac{R_0 \cos l - d}{R}\right) - \Theta_0 \cos l$$

$$\omega = \Theta / R$$

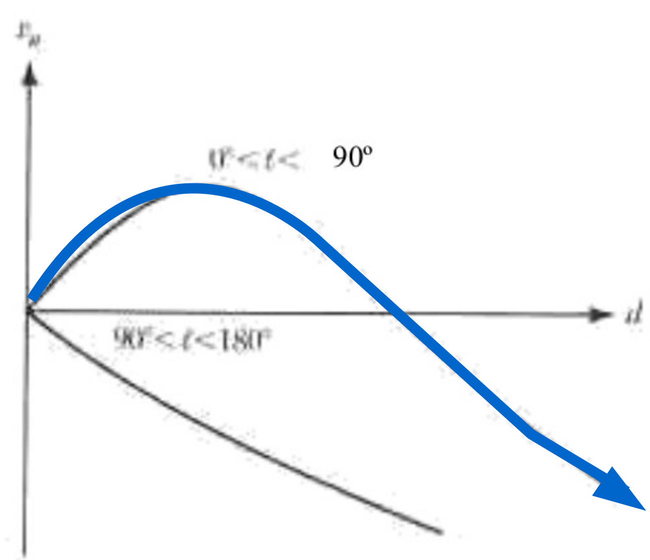
$$v_{POS} = (\omega - \omega_0) R_0 \cos l - \omega d$$



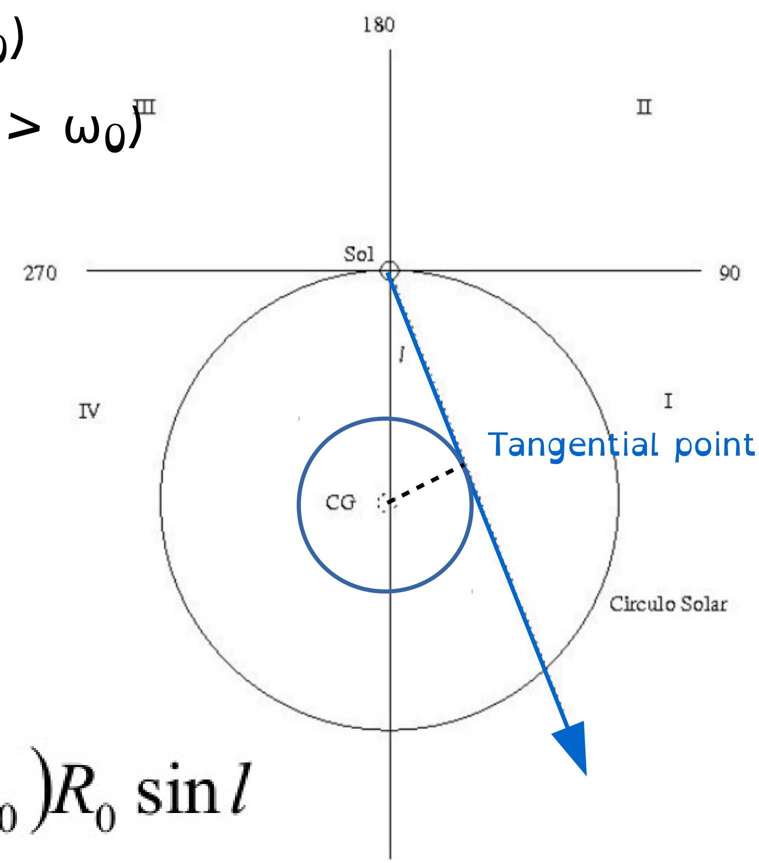
LINE OF SIGHT VELOCITY

AS A FUNCTION OF DISTANCE FROM SUN

- Keep in mind that, for R > 1 kpc, ω monotonically decreases
- Let's suppose to move along a given l, starting from quadrant I:
  - ω grows until the tangential point ($\omega > \omega_0$)
  - ω decreases after the tangential point ($\omega > \omega_0$)
  - at some point $\omega < \omega_0$ (and $v_{LOS} < 0$)



$$v_{LOS} = (\omega - \omega_0)R_0 \sin l$$

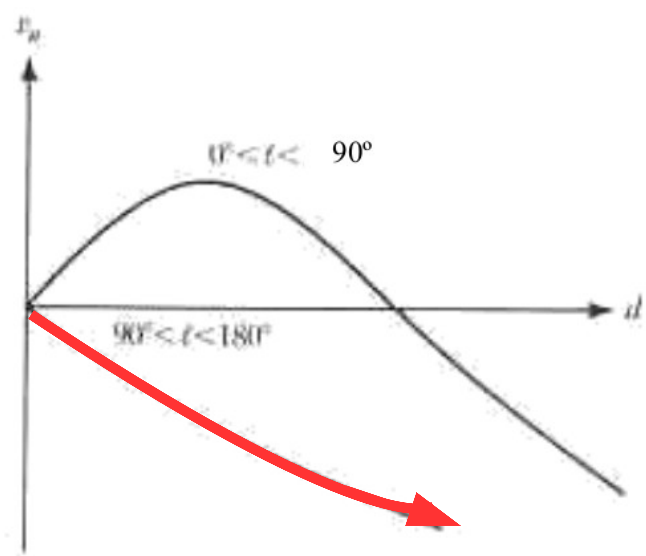


LINE OF SIGHT VELOCITY

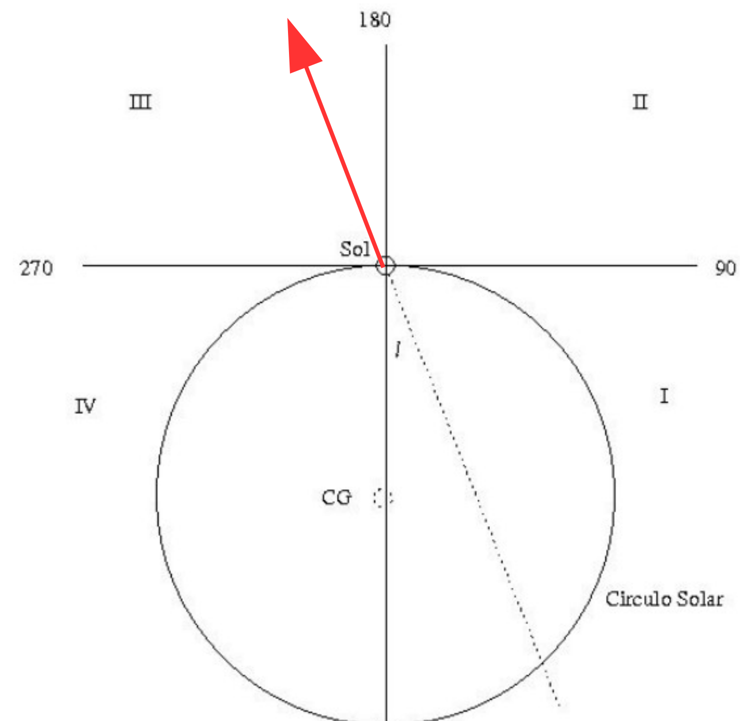
AS A FUNCTION OF DISTANCE FROM SUN

- Keep in mind that, for R > 1 kpc, ω monotonically decreases
- Let's suppose to move along a given *l*, starting from quadrant II:
  - ω monotonically decreases ($\omega < \omega_0$)

    (and v_{LOS} always < 0)

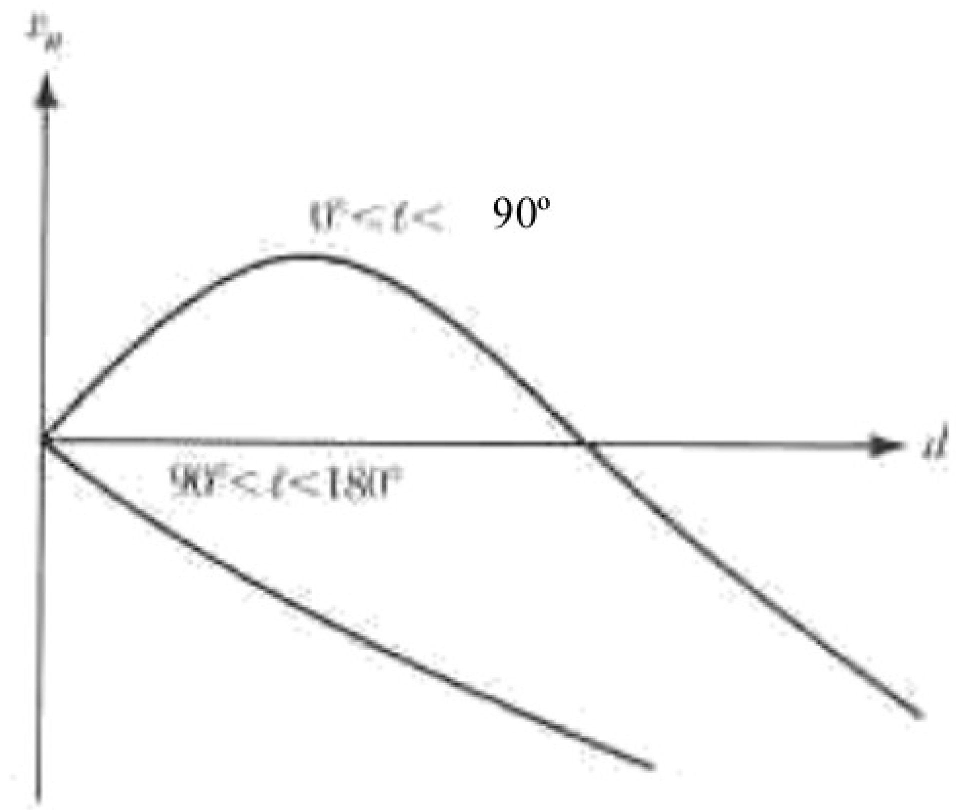


$$v_{LOS} = (\omega - \omega_0) R_0 \sin l$$



LINE OF SIGHT VELOCITY AS A FUNCTION OF DISTANCE FROM SUN

- For the quadrant IV and III, the behavior is like quadrant I and II *(but sign reversed)*



OORT CONSTANTS

- Can we infer the local Galaxy rotation curve from nearby stars?
  → yes if we observe v_{LOS} , v_{POS} , distance and *l*

- We manipulate previous equations to get to a formulation which allows a direct measurement → Oort constants

- NOTE: the Oort approach allowed to confirm and measure the (*local*) differential rotation (1927)

OORT CONSTANTS - V_{LOS}

- We will consider the Solar neighborhood:

  $\omega \sim \omega_0$ ($d < 1$ kpc $<< R_0$)

- Considering the geometry →

  Taylor expansion of ($\omega - \omega 0$) at R0

$$(\omega - \omega_0) \cong \left(\frac{d\omega}{dR}\right)_{R_0} (R - R_0)$$

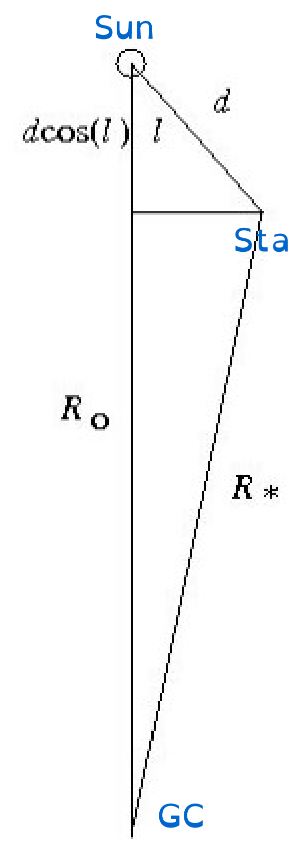
$$R - R_0 \approx -d \cos l$$

$$\omega = \Theta / R$$

$$\frac{d\omega}{dR} = \frac{d}{dR}\left(\frac{\Theta}{R}\right) = \frac{1}{R}\frac{d\Theta}{dR} - \frac{\Theta}{R^2}$$

$$\left(\frac{d\omega}{dR}\right)_{R_0} = \frac{1}{R_0}\left(\frac{d\Theta}{dR}\right)_{R_0} - \frac{\Theta_0}{R_0^2}$$

$$(\omega - \omega_0) = -\left(\frac{1}{R_0}\left(\frac{d\Theta}{dR}\right)_{R_0} - \frac{\Theta_0}{R_0^2}\right) d \cos l$$



OORT CONSTANTS - A

$$v_{LOS} = (\omega - \omega_0) R_0 \sin l \quad \Rightarrow \quad v_{LOS} = -\left(\left(\frac{d\Theta}{dR} \right)_{R_0} - \frac{\Theta_0}{R_0} \right) d \cos l \sin l$$

(seen before)

- We define Oort constant A:

$$A = \frac{1}{2} \left[\frac{\Theta_0}{R_0} - \left(\frac{d\Theta}{dR} \right)_{R_0} \right]$$

- And then, considering that:

$$2 \sin l \cos l = \sin(2l)$$

... we finally write:

$$v_{LOS} = A d \sin(2l)$$

NOTE: In case of rigid rotation dθ/dR = 0 → A = 0

NOTE: We confirm the assumption that V_{LOS} is a sinusoidal function of l

OORT CONSTANTS - V_{POS}

$$v_{POS} = (\omega - \omega_0) R_0 \cos l - \omega d$$

- Taylor expansions:

$$(\omega - \omega_0) = -\left(\frac{1}{R_0} \left(\frac{d\Theta}{dR} \right)_{R_0} - \frac{\Theta_0}{R_0^2} \right) d \cos l$$

(same as before)

$$\omega d = (\cancel{\omega - \omega_0}) d + \omega_0 d.$$

assumption that $\omega \sim \omega_0$

(solar neighborhood)

$$v_{POS} = -R_0 \left(\frac{1}{R_0} \left(\frac{d\Theta}{dR} \right)_{R_0} - \frac{\Theta_0}{R_0^2} \right) d \cos l \cos l - \frac{\Theta_0}{R_0} d$$

$$2 \cos^2 l = (1 + \cos 2l)$$

$$v_{POS} = -\frac{1}{2} \left(\left(\frac{d\Theta}{dR} \right)_{R_0} - \frac{\Theta_0}{R_0} \right) d (1 + \cos 2l) - \frac{\Theta_0}{R_0} d$$

$$v_{POS} = -\frac{1}{2} \left(\left(\frac{d\Theta}{dR} \right)_{R_0} - \frac{\Theta_0}{R_0} \right) d \cos(2l) \quad - \frac{1}{2} \left[\left(\frac{d\Theta}{dR} \right)_{\circ} + \frac{\Theta_{\circ}}{R_{\circ}} \right] d$$

OORT CONSTANTS - B

$$v_{POS} = -\frac{1}{2}\left(\left(\frac{d\Theta}{dR}\right)_{R_0} - \frac{\Theta_0}{R_0}\right) d\cos(2l) \quad -\frac{1}{2}\left[\left(\frac{d\Theta}{dR}\right)_{\circ} + \frac{\Theta_{\circ}}{R\circ}\right] d$$

- We define Oort constant B:

$$B = -\frac{1}{2}\left[\frac{\Theta_0}{R_0} + \left(\frac{d\Theta}{dR}\right)_{R_0}\right]$$

- And then, we finally write:

$$v_{POS} = Ad\cos 2l + Bd$$

NOTE: Even in case of rigid rotation θ_0/R_0 != 0 → B != 0

OORT CONSTANTS - CONSIDERATIONS

- Summary:

$$v_{POS} = Ad\cos 2l + Bd \qquad v_{LOS} = Ad\sin(2l)$$

where:

$$A = \frac{1}{2}\left[\frac{\Theta_0}{R_0} - \left(\frac{d\Theta}{dR}\right)_{R_0}\right] \qquad B = -\frac{1}{2}\left[\frac{\Theta_0}{R_0} + \left(\frac{d\Theta}{dR}\right)_{R_0}\right]$$

→ so we can obtain A and B by measuring v_{LOS} , v_{POS} , d, and *l*

- The Oort constants directly give:

$$A - B = \frac{\Theta_0}{R_0} = \omega_0$$

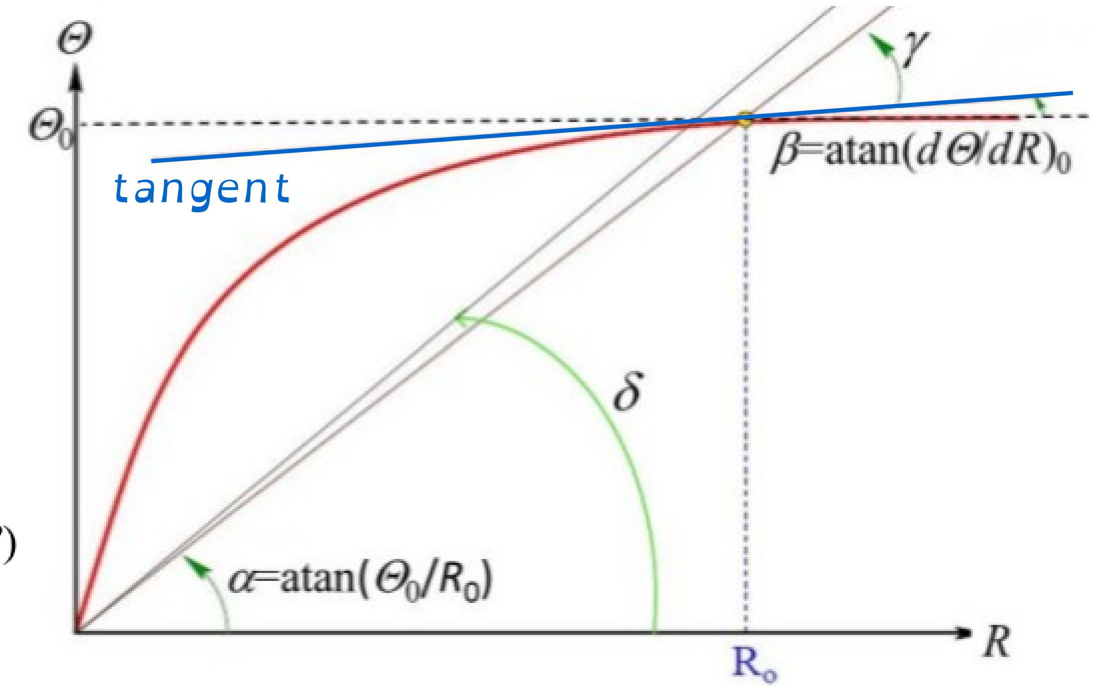
→ *local* rotational velocity

$$A + B = -\left(\frac{d\Theta}{dR}\right)_{R_0}$$

→ *local* velocity gradient (shear)

OORT CONSTANTS - INTERPRETATION W/R TO VELOCITY CURVE

- Let's assume an hypothetical rotation curve (quite realistic):
  - β → tangent angle
  - α → angle to point
  - δ = α + β
  - γ = α – β



$$A = \frac{1}{2}[\tan\alpha - \tan\beta] = \tan\gamma(1+\tan\alpha\tan\beta)$$

$$B = -\frac{1}{2}[\tan\alpha + \tan\beta] = \tan\delta(1-\tan\alpha\tan\beta)$$

→ the Oort constants help constraining the functional form of the Galaxy rotation curve (by studying the local neighborhood) !

OORT CONSTANTS - PROPER MOTION

- Let's write the proper motion μ ["/sec] with Oort constants

$$v_l = \mu_l \cos(b) d$$

tangential velocity (previous class)

- For a motion in the galactic disk (b=0), the assumptions of the Oort constants are valid, and we can say:

$$v_l = \mu_l\ d = v_{POS}$$

$$\mu_l = v_{POS} / d$$

If using units of [km/sec] and ["/year]:

$$\mu_l = \frac{v_l}{4.74d}$$

$$\mu_l = \frac{A \cos 2l + B}{4.74}$$

OORT CONSTANTS - MEASUREMENTS

- We can get A and B from:

measurables

$$v_{POS} = Ad\cos 2l + Bd \qquad v_{LOS} = Ad\sin(2l)$$

	Hipparchos	Gaia
A	14.8 (±0.8) km/s/kpc	15.3 (±0.4) km/s/kpc
B	-12.4 (±0.6) km/s/kpc	-11.9 (±0.4) km/s/kpc
A – B (ω_0)	27.2 km/s/kpc	27.2 km/s/kpc
A + B ($-d\Theta/dR \mid R_0$)	2.4 km/s/kpc	3.4 km/s/kpc

NOTES:

- $\frac{\Theta_0}{R_0} = \omega_0$ = (A-B) & Θ_0 ~ 220 km/sec → R_0 ~ 8 kpc (while used 8.5 km)

Hipparchos: Feast & Whitelock (1997)

Gaia data: Bovy 2017, MNRAS, 468, 63

OORT CONSTANTS - FINAL REMARK

- A more accurate Taylor expansion, and relaxing the b = 0 assumption leads to more constants:

$$
\begin{aligned}
2A &= \quad \bar{v}_{\phi}/R_0 - \bar{v}_{\phi,R} - \bar{v}_{R,\phi}/R_0 \\
2B &= -\bar{v}_{\phi}/R_0 - \bar{v}_{\phi,R} + \bar{v}_{R,\phi}/R_0 \\
2C &= -\bar{v}_{R}/R_0 + \bar{v}_{R,R} - \bar{v}_{\phi,\phi}/R_0 \\
2K &= \quad \bar{v}_{R}/R_0 + \bar{v}_{R,R} + \bar{v}_{\phi,\phi}/R_0
\end{aligned}
$$

Bovy 2017, MNRAS, 468, 63

2 – ROTATION CURVE

- To study the rotation curve we will use the ISM, in particular:
  - HI (*neutral*)
  - CO (tracing H2 – *we will see it later*)
- ISM has a larger distribution than stars, hence it traces the rotation to larger radii

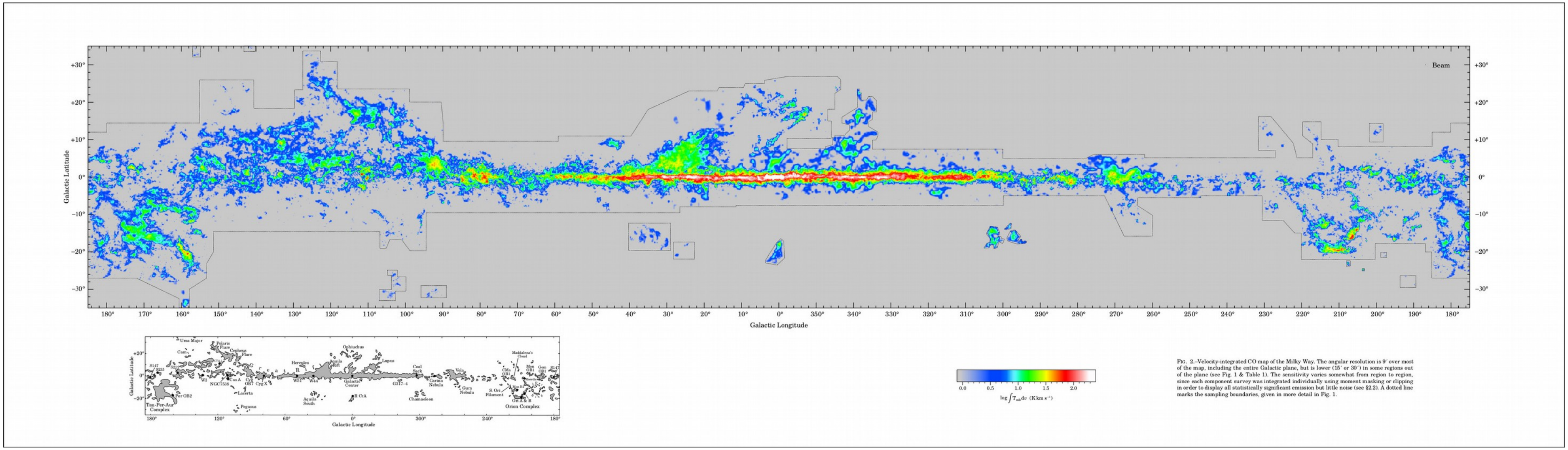


FIG. 2.–Velocity-integrated CO map of the Milky Way. The angular resolution is 9′ over most of the map, including the entire Galactic plane, but is lower (15′ or 30′) in some regions out of the plane (see Fig. 1 & Table 1). The sensitivity varies somewhat from region to region, since each component survey was integrated individually using moment masking or clipping in order to display all statistically significant emission but little noise (see §2.2). A dotted line marks the sampling boundaries, given in more detail in Fig. 1.

[Dame, Hartmann, and Thaddeus (2001)]

POSITION-VELOCITY DIAGRAM

- One way to look at the rotation curve is using the (v,*l*) diagram

  Compressing the b coordinate around |b| < 2°

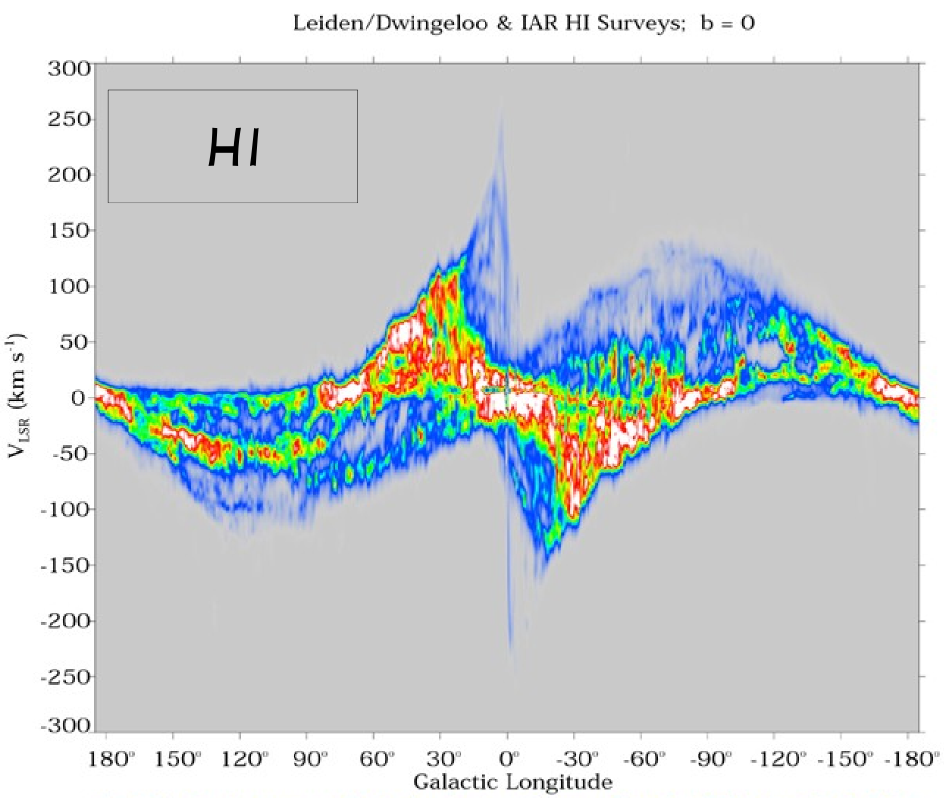
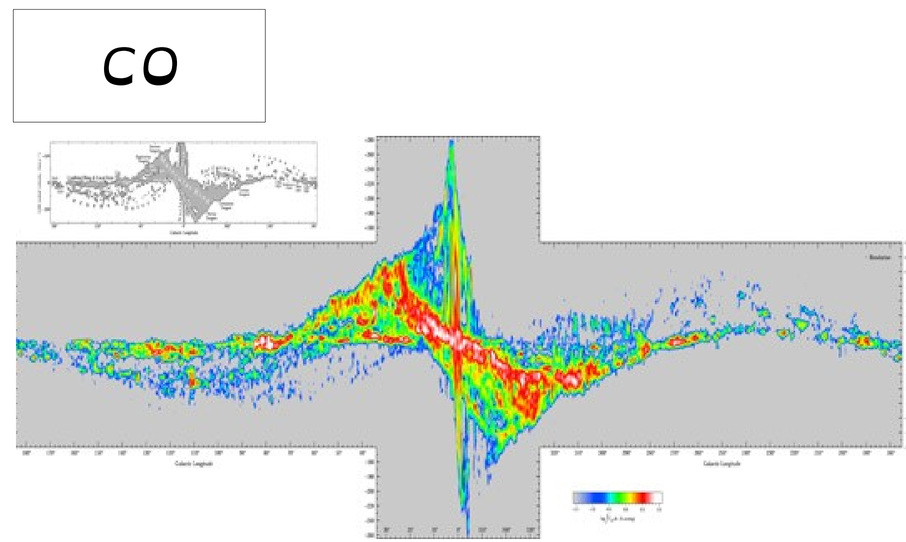


Fig 2.20 (D. Hartmann) 'Galaxies in the Universe' Sparke/Gallagher CUP 2007



UNDERSTANDING THE POSITION-VELOCITY DIAGRAM

- Let's understand the shape of the rotation curve
- Remember the expression for v_{POS}:

$$v_R = (\omega - \omega_0) R_0 \sin l$$

→ for each concentric gas ring, ω = CONST → $v_R \sim \sin(l)$

→ each ring appears as a sinusoidal curve

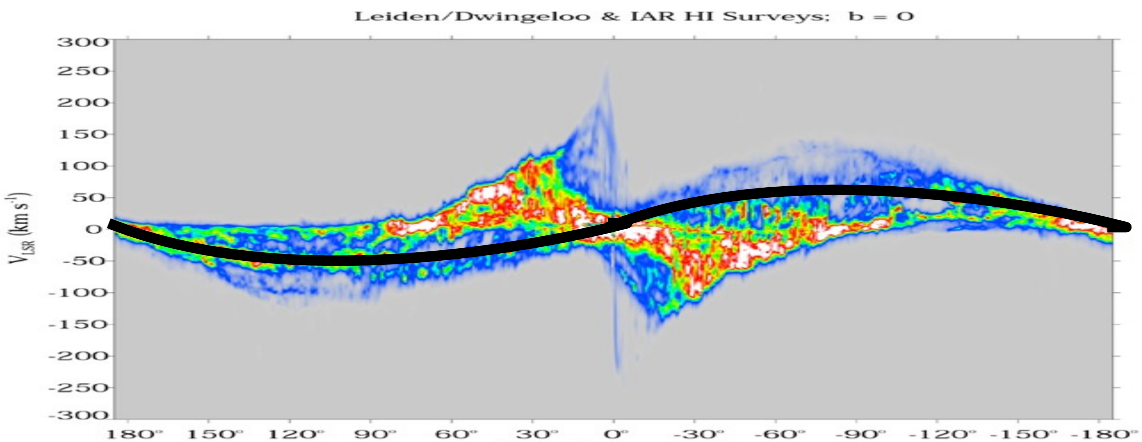


Fig 2.20 (D. Hartmann) 'Galaxies in the Universe' Sparke/Gallagher CUP 2007

UNDERSTANDING THE POSITION-VELOCITY DIAGRAM – OUTER ORBITS

- Outer rings are visible at all l (-180° < l < 180°)
  - velocities $\omega = \omega_0$ to ω_{MAX} (always $\omega < \omega_0$)

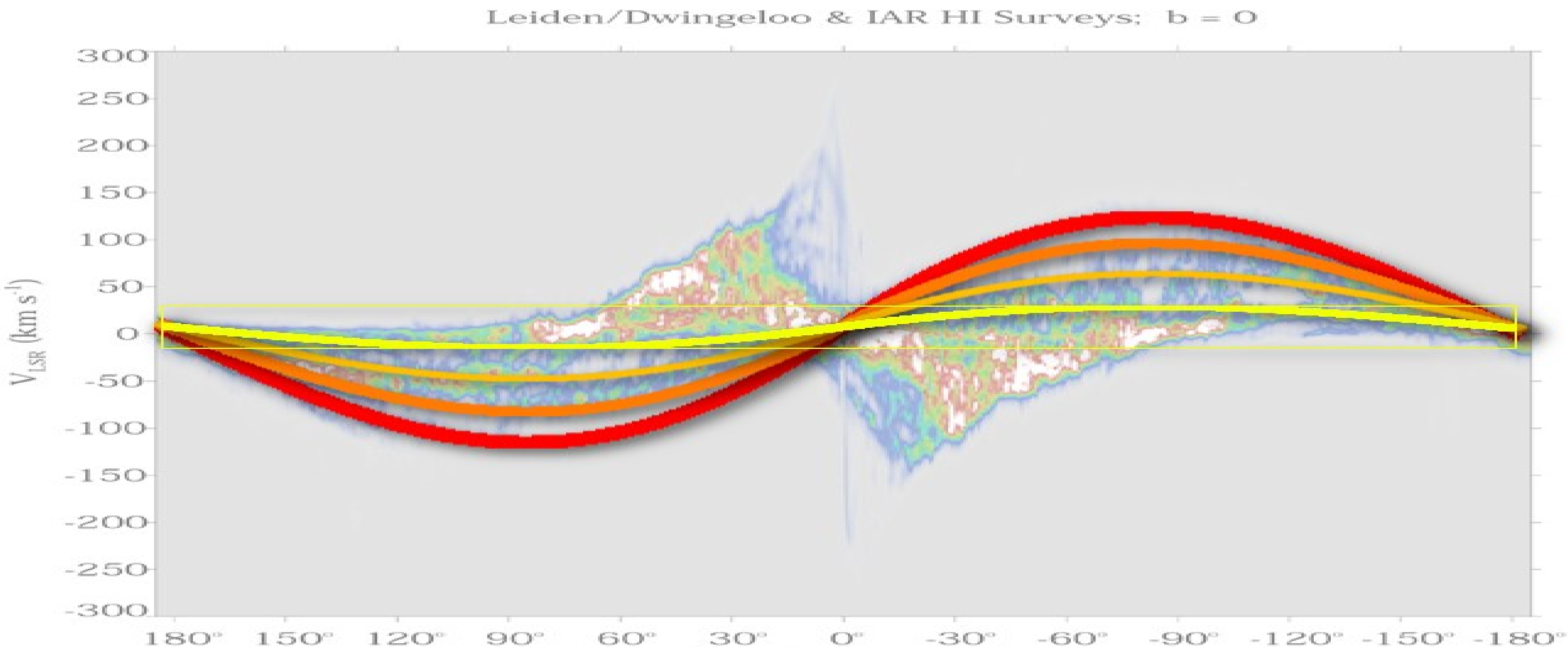
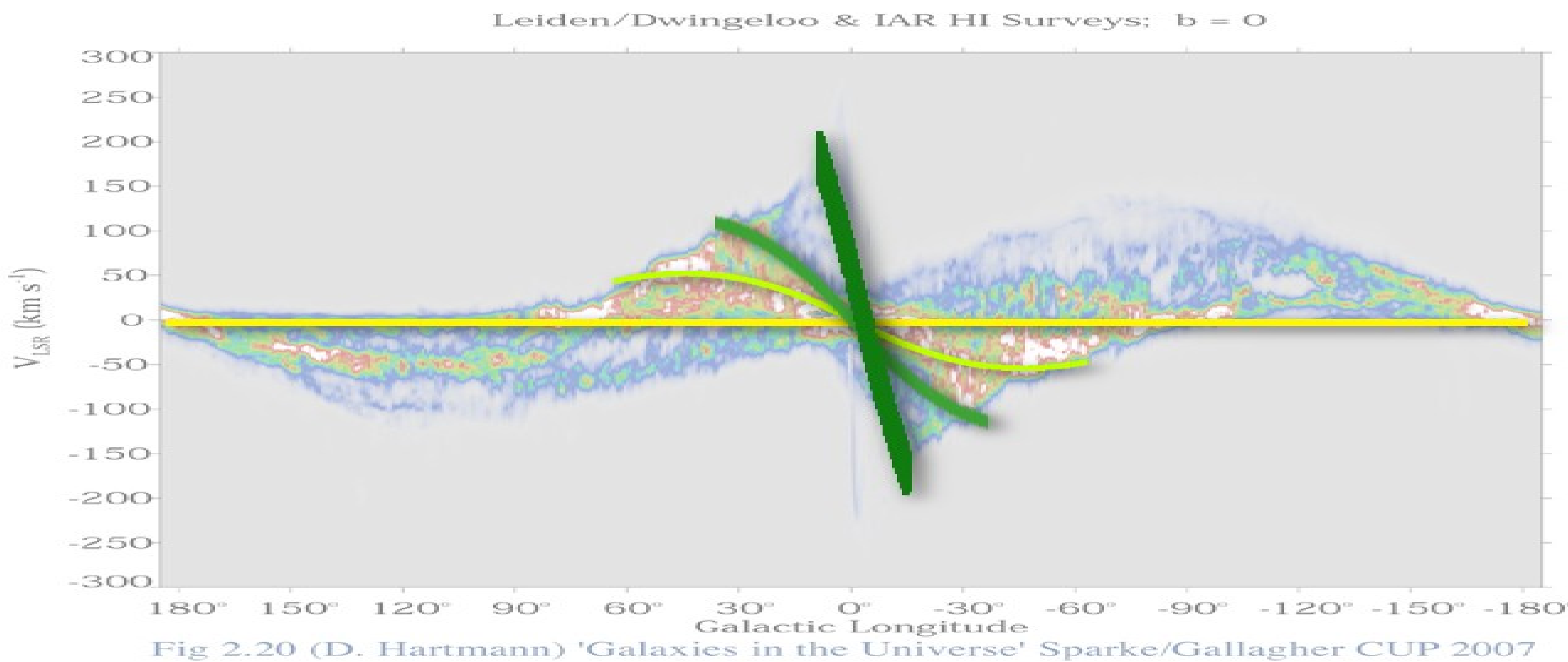


Fig 2.20 (D. Hartmann) 'Galaxies in the Universe' Sparke/Gallagher CUP 2007

UNDERSTANDING THE POSITION-VELOCITY DIAGRAM – INNER ORBITS

- Inner rings are visible between $l_{min} < l < l_{MAX}$
  - velocities $\omega = 0$ to $\omega = \omega_0$ (always $\omega > \omega_0$)



UNDERSTANDING THE POSITION-VELOCITY DIAGRAM – INNER ORBITS

- Inner rings are visible between $l_{min} < l < l_{MAX}$

NOTE: intensity of inner orbits is stronger ← crossed twice along LOS

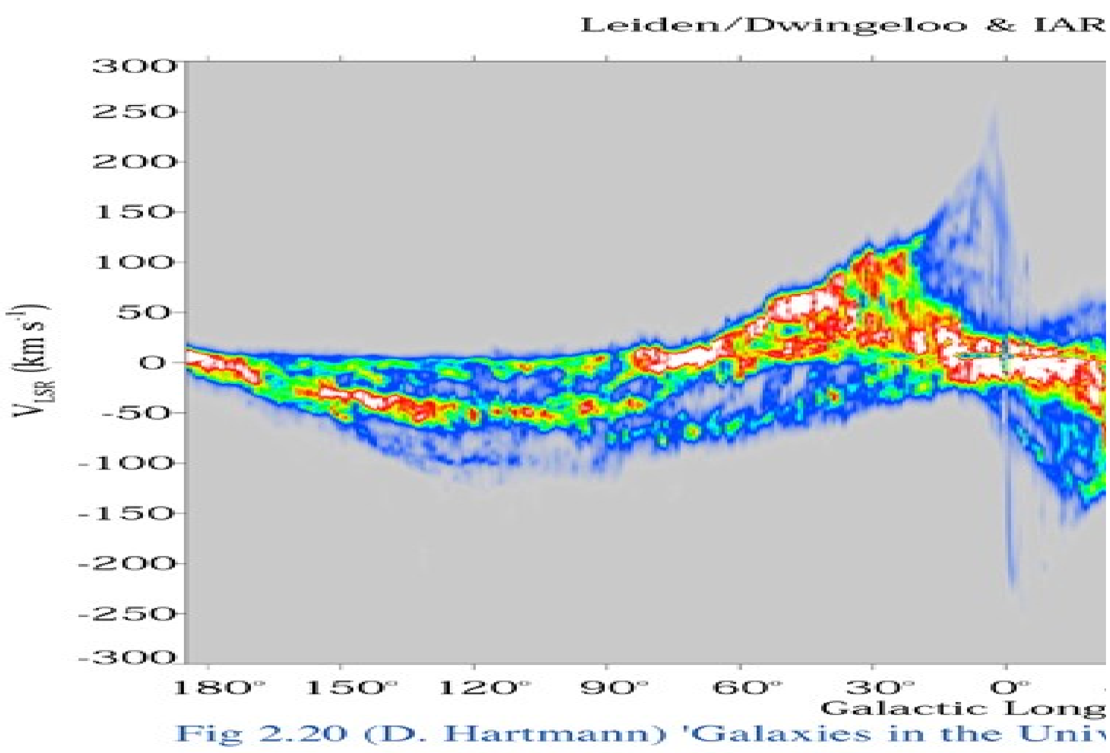
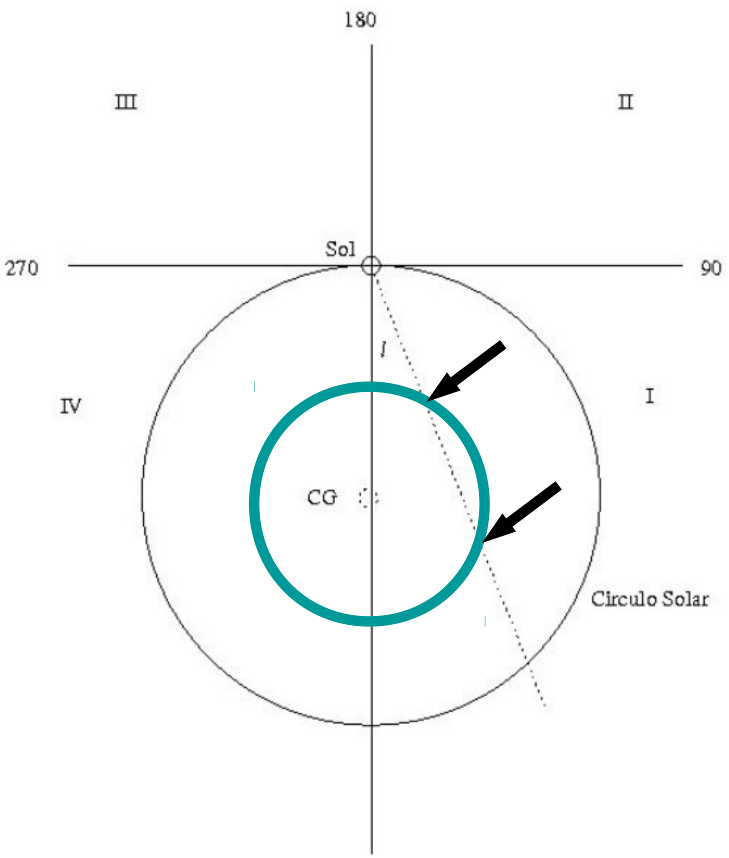


Fig 2.20 (D. Hartmann) 'Galaxies in the Uni



UNDERSTANDING THE POSITION-VELOCITY DIAGRAM – SUB-STRUCTURE

- Additional sub-structure is due to spiral arms/bar (see later)

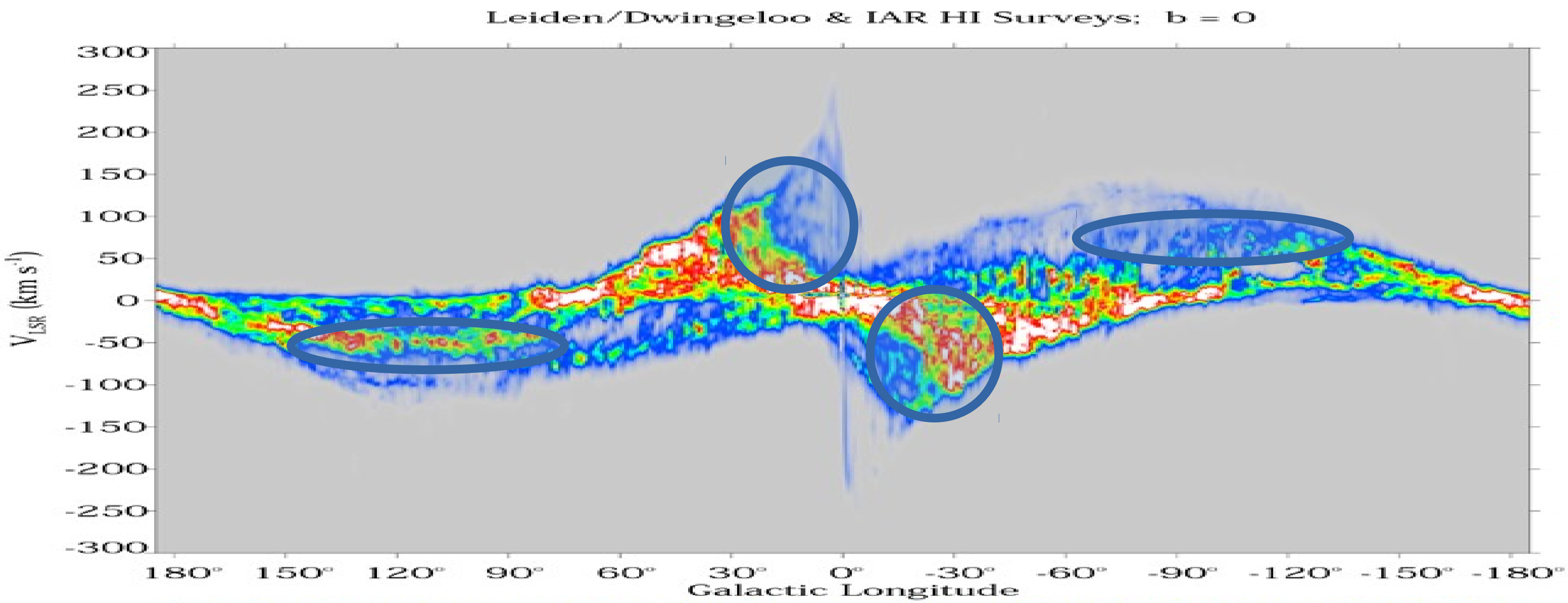


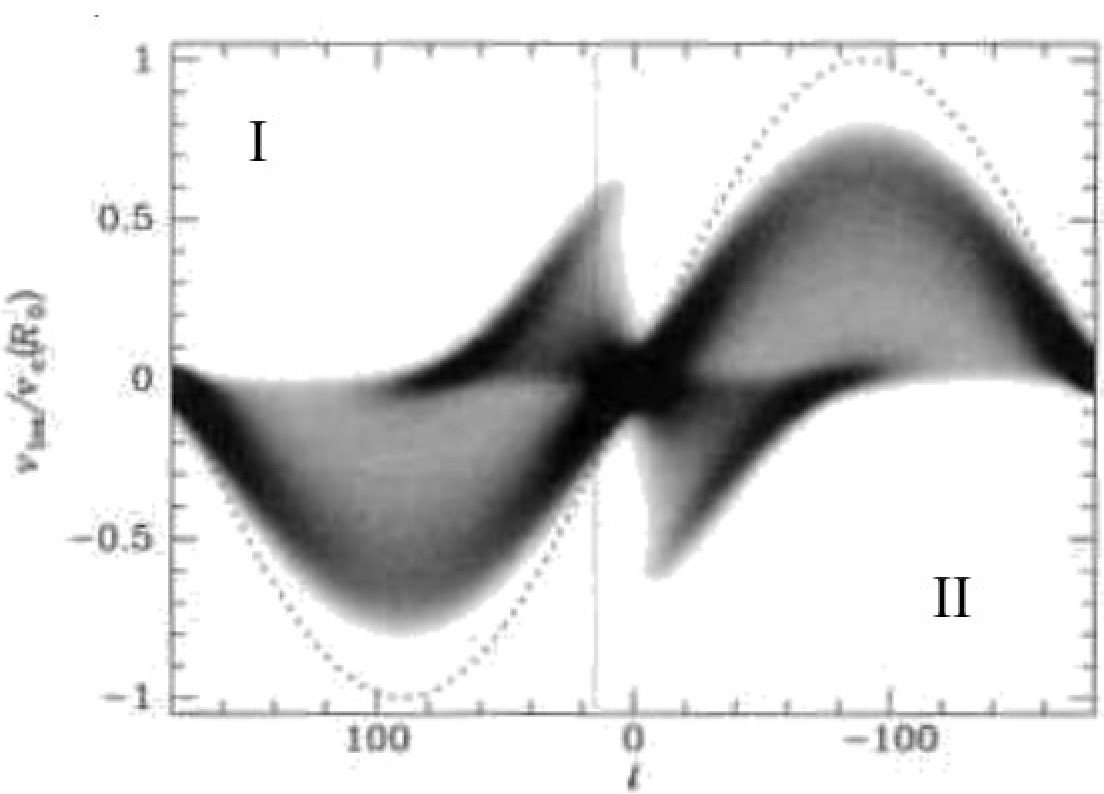
Fig 2.20 (D. Hartmann) 'Galaxies in the Universe' Sparke/Gallagher CUP 2007

THEORETICAL POSITION-VELOCITY DIAGRAM

- Theoretical prediction of a disk gas distribution rotating as:

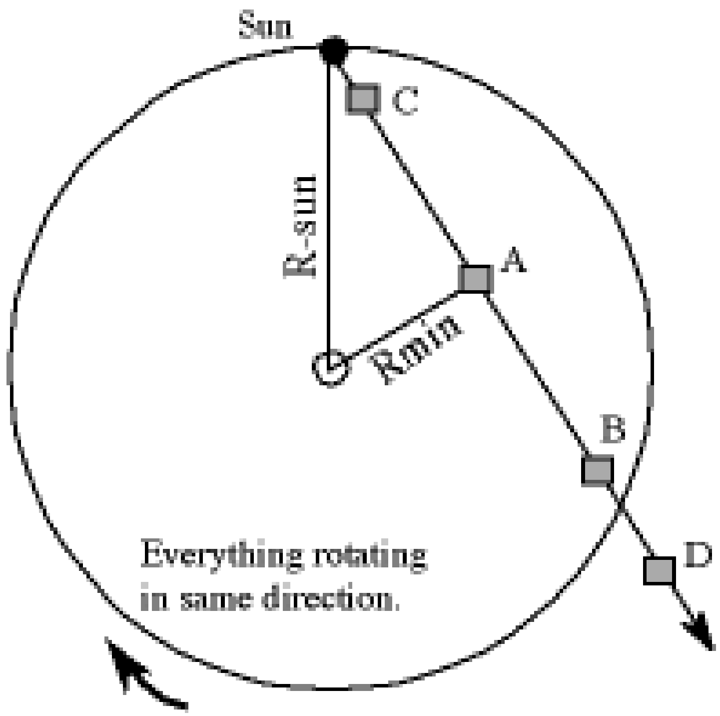
$$v_R = (\omega - \omega_0) R_0 \sin l$$

→ close to real distribution! *(more on this later)*



VELOCITY OF INNER MOLECULAR CLOUDS

- There are several clouds along a given LOS, with different ω
  – how we can associate a radius to the velocity?

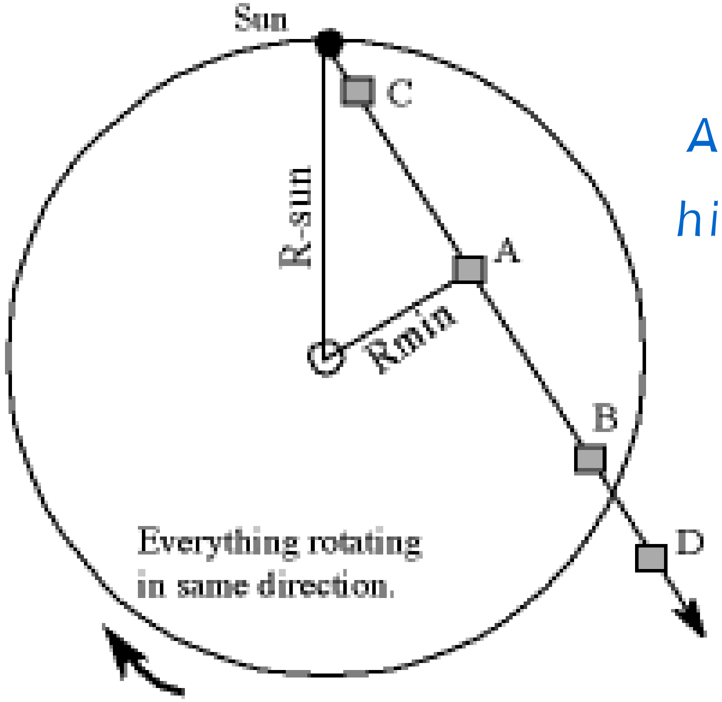


- If we obtain a spectrum along the LOS, we will see peaks of emission at different redshift

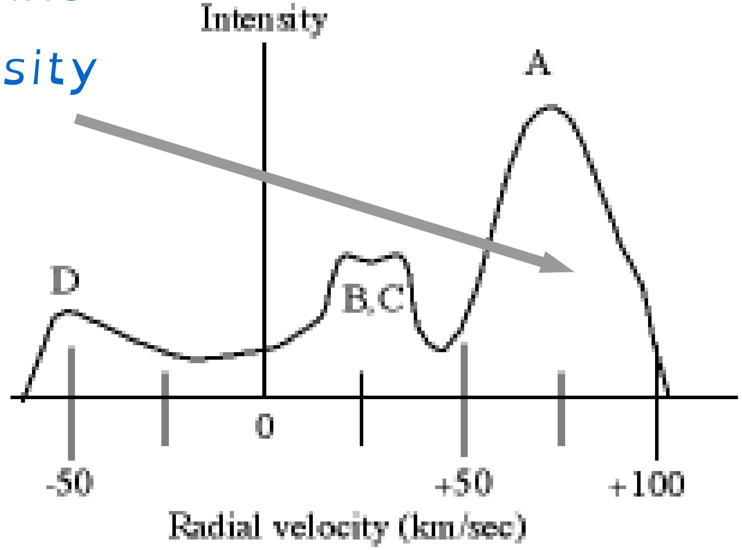
VELOCITY OF INNER MOLECULAR CLOUDS

- We can convert the redshift into radial velocity
- For cloud A:
  - $v_{LOS} = v$
  - v_{LOS} is MAX ($v_{LOS,MAX}$ closer to center)

$\rightarrow \omega \leftrightarrow R = R_0 \sin(l)$

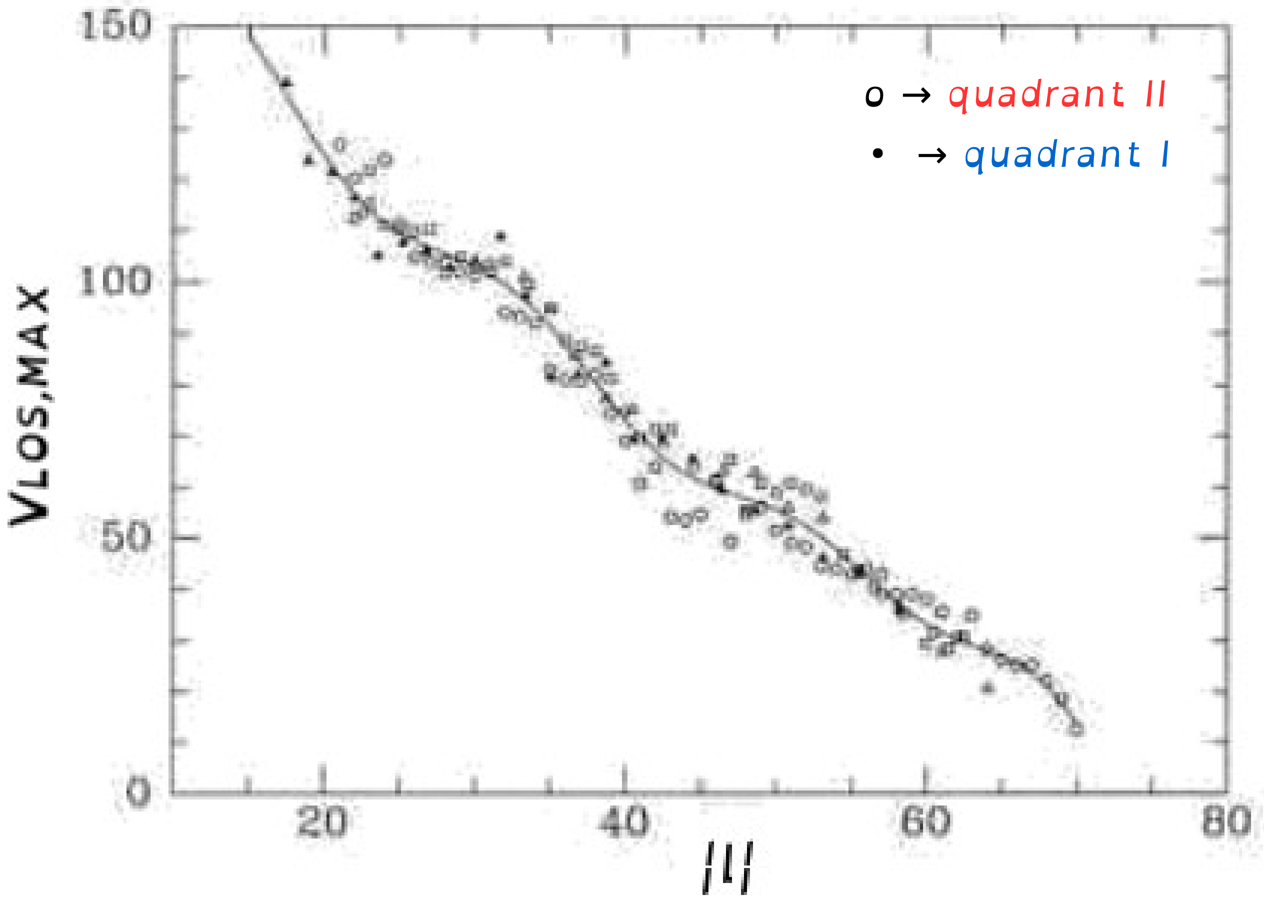


A has also the highest density



ROTATION OF INNER MOLECULAR CLOUDS

- … by repeating at different l, we get the *inner* $v_{LOS,MAX}$ (= v):



SWITCHING TO ROTATION CURVE

- To get to the actual rotation curve (θ VS. R), we need to:
  - use R_{min} instead of l
  - convert v (LSR_sun) to θ (SFR)

General conversion formula (see before):

$$v_R = \Theta \cos\alpha - \Theta_0 \sin l$$

for $v_{LOS,MAX}$, $\cos(\alpha) = 1$:

$$v_{\max} = \Theta - \Theta_0 \sin l$$

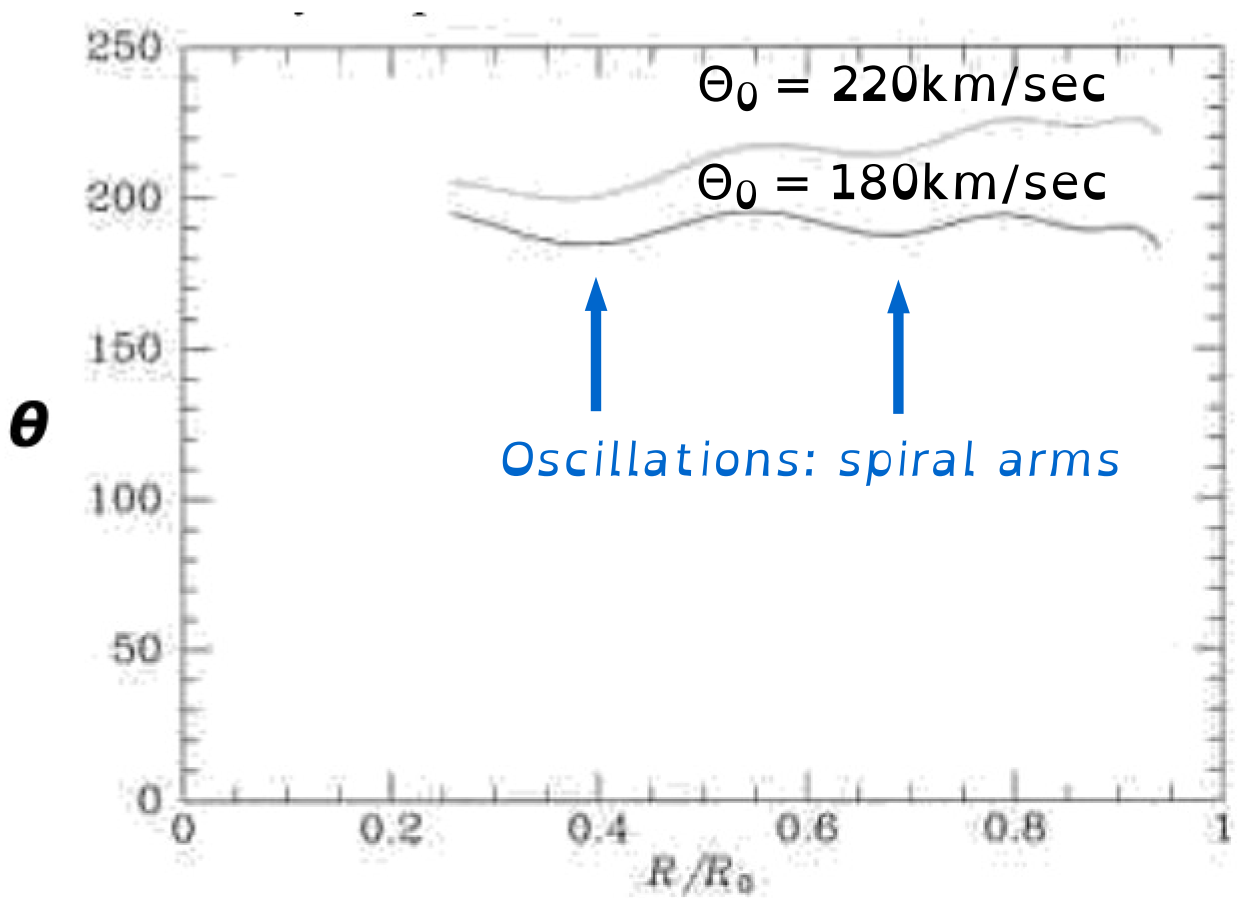
Finally:

$$\theta = v_{LOS,MAX} + \theta_0 \sin(l)$$

→ the normalization will depend on θ_0

INNER ROTATION CURVE

- This way, we get the [2-to-8 kpc] rotation curve:



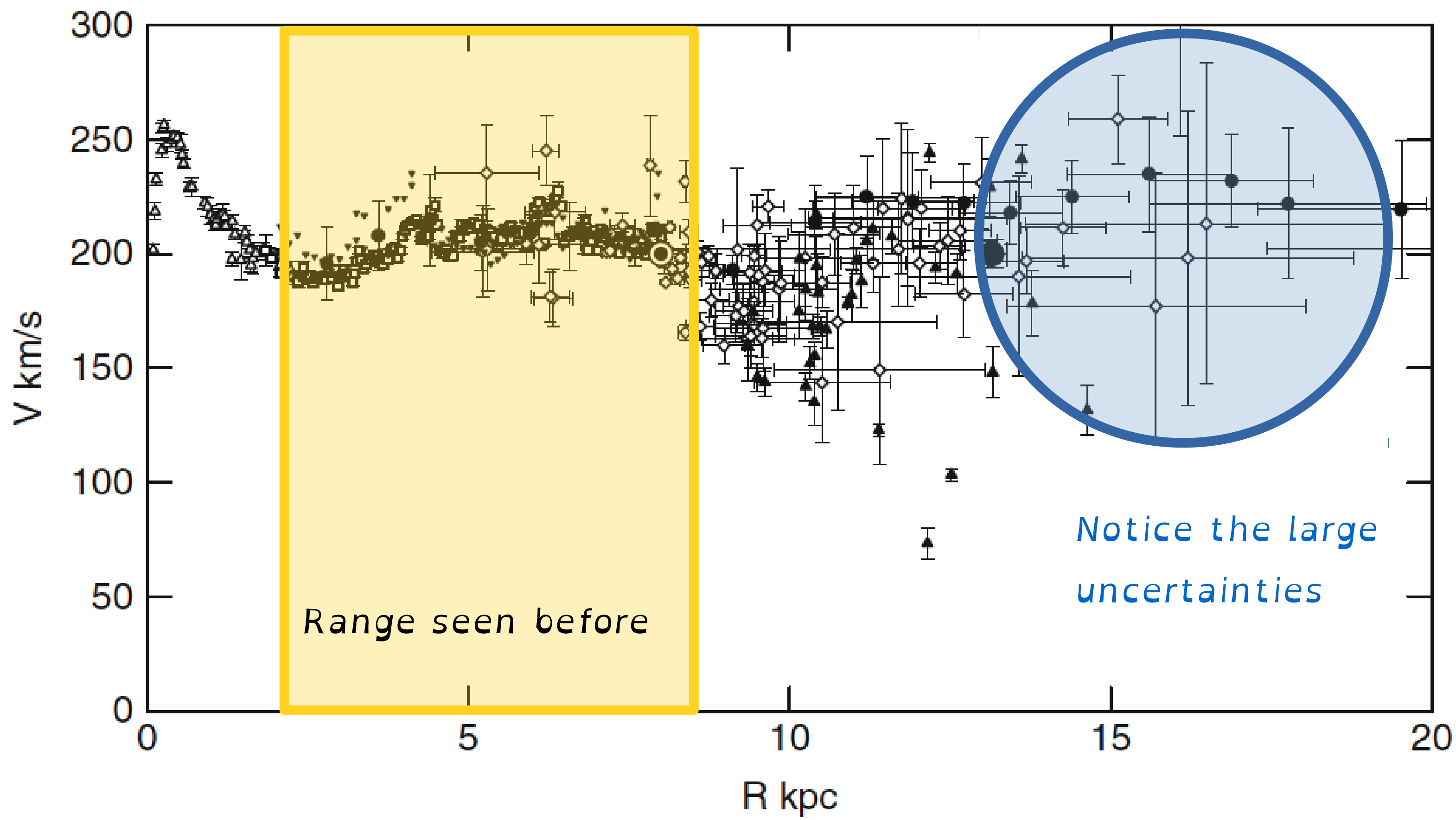
OUTER ROTATION CURVE

- Outside the solar circle, we cannot use the same technique

- We need to use objects for which we can measure both:
  - distance
  - velocity
- Possible objects:
  - Cepheids
  - Planetary/HII nebulae of “known” size
  - Gas associated with young clusters
    → gas radio emission yields velocity
    → main-sequence fitting yields distance modulus
- All these are affected by large uncertainties

FULL ROTATION CURVE

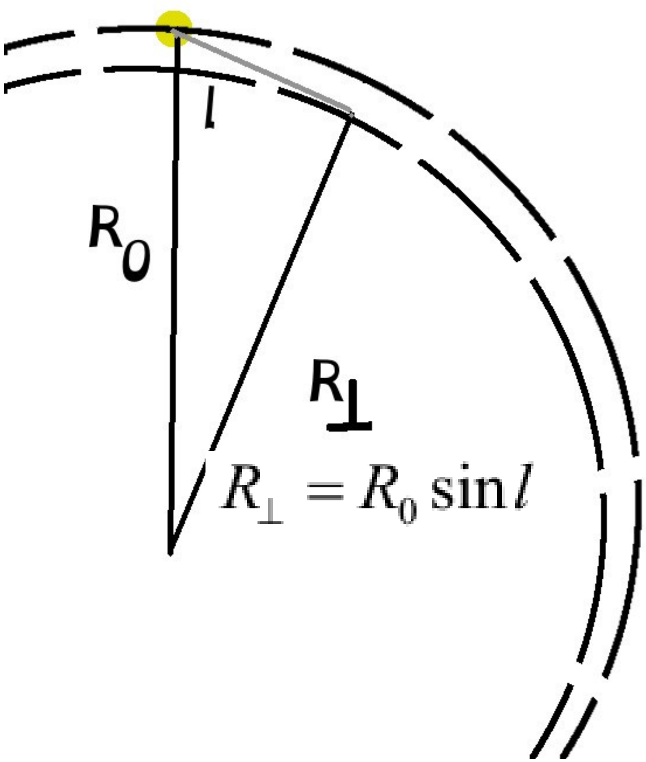
- Et volià:



[Sofue et al. 2009, PASJ, 61, 227]

ROTATION CURVE –
CONSTRAINTS BY OORT CONSTANT

- Let's do an exercise: use the Oort constants to constrain the rotation curve derived with the method above



$$v_{LOS} = v_\perp = \Theta(R_\perp) - \Theta_0 \sin l$$

In the solar neighborhood: $R_0 \sim R_\perp$

→ we can perform the Taylor expansion:

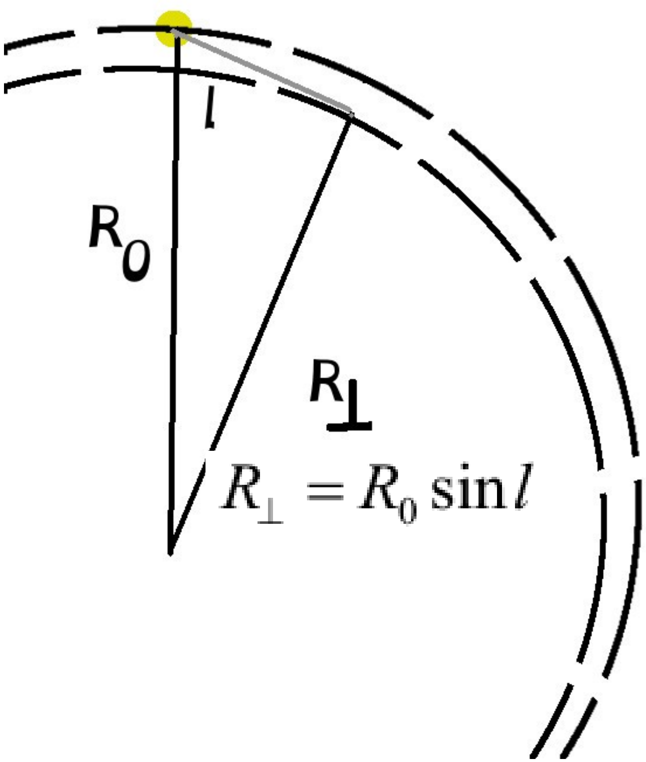
$$\Theta(R_\perp) \approx \Theta_0 + \left(\frac{d\Theta}{dR}\right)_{R_\circ} (R_\perp - R_\circ) + O(2)$$

And it holds:

$$R_\perp - R_0 = -R_\circ(1 - \sin l)$$

ROTATION CURVE – CONSTRAINTS BY OORT CONSTANT

- Let's do an exercise: use the Oort constants to constrain the rotation curve derived with the method above



So we can write:

$$v_{LOS} = v_\perp = \Theta(R_\perp) - \Theta_0 \sin l$$

as:

$$v_\perp \approx \left[\frac{\Theta_0}{R_0} - \left(\frac{d\Theta}{dR} \right)_{R_\circ} \right] R_0 \; (1 - \sin l) + O(2)$$

$$v_\perp \approx 2AR_\circ (1 - \sin l) + O(2)$$

→ measuring v_{LOS} we obtain info about AR_0

i.e. about the rotation curve

MEASURING DISTANCE FROM GAS CLOUDS

- How to measure distance d from a gas cloud?

  Differently from stars, the intrinsic L of a cloud is practically impossible to estimate (depends on T, τ, etc.)

  → cannot measure distance modulus

- Alternatives:

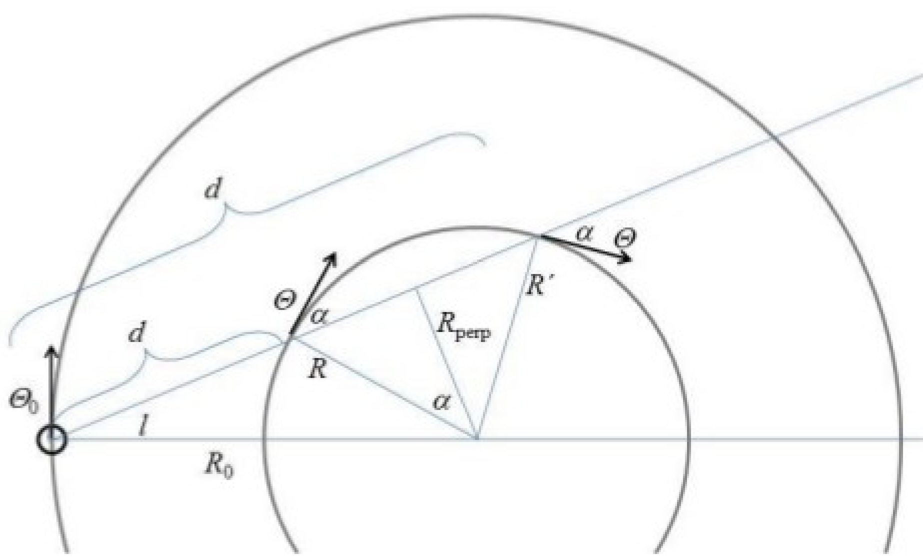
  - measure absorption of a background star

  - assume a rotation curve $\Theta(R)$ and $\omega(R)$, and circular orbits

MEASURING DISTANCE FROM GAS CLOUDS – ROTATION CURVE

- For an inner cloud, there are two possible distances along LOS:

→ we need R



- We know from before that:

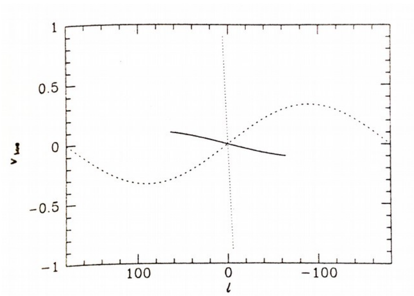
$$v_{LOS} = (\omega - \omega_0) R_0 \sin l$$

where ω = ω(R) is known, and is monotonic with R (for R > 0.5 kpc)

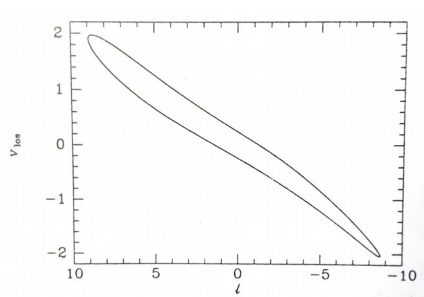
→ there is only one value of R which solves the equation *(we can solve it numerically)*

SPIRAL STRUCTURE FROM GAS ROTATION - NON-CIRCULAR ORBITS

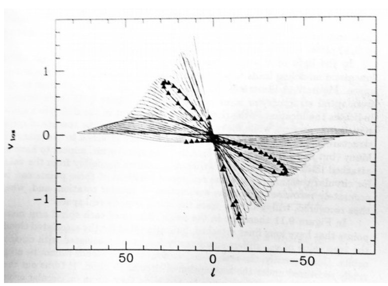
- Non-circular orbits in the (v,*l*) diagram



- From Galactic Astronomy, chap.9.1.1 on on:
  - → circular orbits (inner and outer)



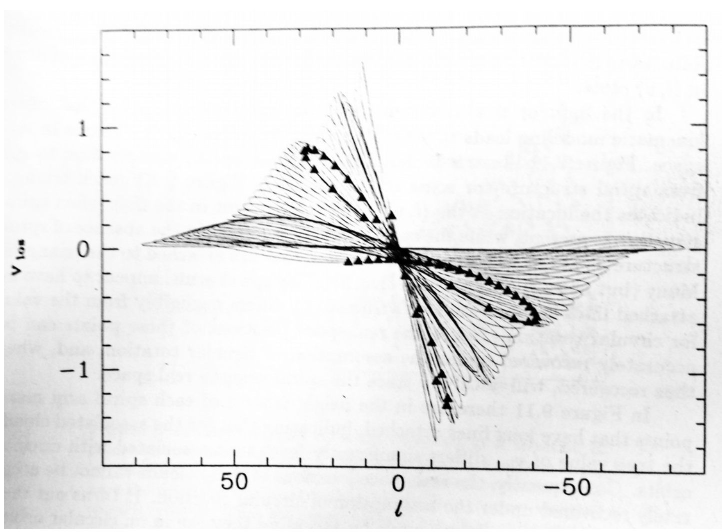
→ elliptical orbit ~ closed inner orbit

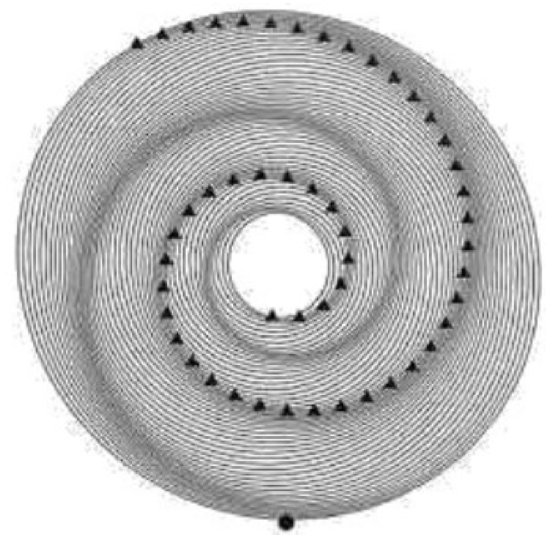


→ spiral orbit ~ non-closed elliptical orbit

SPIRAL STRUCTURE FROM GAS ROTATION

- As we saw before, we can trace any point on (v,l) diagram into R
  → features in the (v,l) reflect into space (assuming circular orbits)

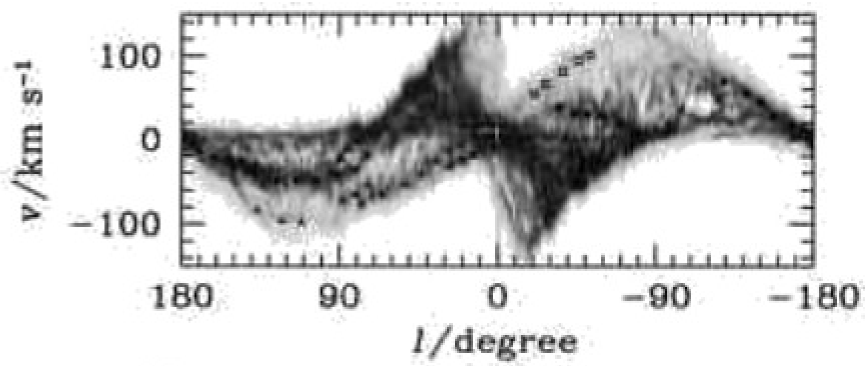


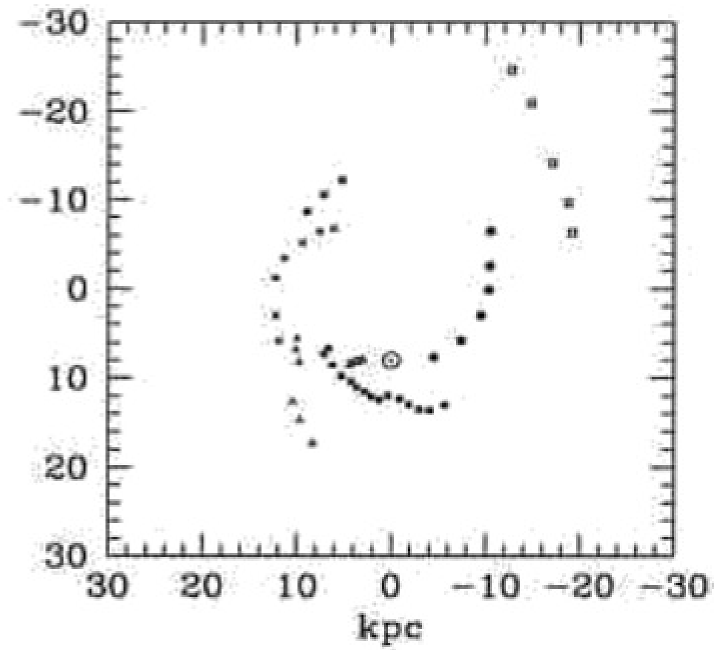


simulation

SPIRAL STRUCTURE FROM GAS ROTATION

- As we saw before, we can trace any point on (v,*l*) diagram into R
  → features in the (v,*l*) reflect into space (assuming circular orbits)





real gas data

SPIRAL STRUCTURE FROM GAS ROTATION

- HI gas map of the Milky Way (behind the nucleus is obscured)