Yukawa Theories as Effective Theories of Quantum Chromodynamics for a Large Number of Colors

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An effective theory for low-energy nuclear interactions is proposed, based on results obtained from the $1/N_c$ expansion of quantum chromodynamics. The Lagrangian is local in the meson sector, but in the baryon sector it is nonlocal both in the meson-baryon Yukawa coupling and in the baryon propagators. It satisfies an important consequence of the $1/N_c$ expansion, the suppression of baryon loops. Our findings are then shown to support the traditional approaches in nuclear physics and, more especially, the relativistic nuclear many-body theories at baryon-tree level.

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Relativistic nuclear many-body theories[1,2] have shown surprisingly impressive, phenomenological success. Though some underlying physics is not completely understood,[3] these theories, together with the Bonn *NN* potential[4] and Dirac phenomenology,[5] have revealed interesting relativistic degrees of freedom in nuclei.

Their success has been demonstrated by the use of the mean-field approximation. Recently, however, it has become fashionable to examine the nuclear effects of radiative corrections of these Yukawa theories.[6,7] How the Yukawa theories should be applied in nuclear problems as field theories is an old problem. It is a problem because, though renormalizable, the theories are not asymptotically free and possess no known ultraviolet fixed points.[8] Furthermore, the β function evaluated from one-loop diagrams indicates that perturbative expansions will fail near the nucleon mass scale where the coupling constants become uncontrollably large.[9]

In this Letter, we examine the problem by demanding that the theories be effective theories of quantum chromodynamics (QCD) and that they be applied in a way consistent with it.[10] We use a unified approach based on a rigorously controlled approximation,[11] $1/N_c$ expansion, where N_c is the number of colors. We find that the traditional approach in nuclear physics and, more especially, the mean-field application of the relativistic many-body theories are indeed consistent with QCD in the large-N_c limit. This itself is perhaps no surprise, but we also find that the fashionable calculations of baryon loops *explicitly* contradict QCD in this limit. This finding seems to be supported by recent baryon-loop calculations.[7]

Except for the lattice QCD, the $1/N_c$ expansion is the only possibly effective method in the nonperturbative QCD for low-energy phenomena. There is no decisive test of the method, but the previous theoretical and phenomenological works[12-15] have amply demonstrated that it provides fruitful results. As in these theoretical works, we carefully apply the large-N_c scheme together with ideas of the renormalization group. Naturally, we do not claim our results to be thoroughly rigorous; otherwise, we would have solved QCD in the low-energy regime.

Our basic strategy is as follows: We first construct the effective Lagrangian of the large-N_c limit of QCD with care so as to incorporate all the known symmetries of QCD. We then investigate the consistency of the perturbative expansion of the Lagrangian with its $1/N_c$ expansion. *At each step, we compare a hadronic diagram to corresponding QCD diagrams to ensure consistency.* This comparison is the vital part of our strategy and has been used to test all of our discussions in this work.

Let us first examine a simpler case in the well-studied meson sector.[16] In the large-N_c limit of QCD with the confinement assumption, the meson interactions are described as local, and the meson n-point functions contribute as $O(N_c^{1-n/2})$ in the leading order.[13,14] The meson Lagrangian is then written as

$$\mathcal{L}_m = \tfrac{1}{2}\partial_\mu\phi\,\partial^\mu\phi - \tfrac{1}{2}m^2\phi^2 + \frac{\lambda_1}{\sqrt{N_c}}\phi^3 + \frac{\lambda_2}{N_c}\phi^4 + \frac{\lambda_3}{N_c^{3/2}}\phi^5 + \frac{\lambda_4}{N_c^2}\phi^6 + \cdots, \tag{1}$$

where the N_c dependence of the couplings is explicitly shown; i.e., $m,\lambda_1,\lambda_2,\ldots = O(1)$, here and throughout this Letter. As a continuum field theory, Eq. (1) is not renormalizable and requires a cutoff. The method of the renormalization group dictates higher-derivative (i.e., of higher-mass dimensions) terms to be irrelevant in the continuum limit, and thus these terms are suppressed. ϕ can be any meson field, provided that some interaction terms are discarded to satisfy the symmetry properties associated with each meson and to avoid double counting among the terms. QCD for large N_c does impose some restrictions on ϕ, however. (1) Chiral-symmetry breaking occurs[17] in the leading order, so the flavor symmetry is spontaneously broken to an anomaly-free subgroup, $U(N_f)_R\times U(N_f)_L \to U(N_f)_V$. The hadron spectrum is parity doubled except for Goldstone bosons which are parity odd. (2) In the next-to-leading order, the $U(1)_A$

Goldstone boson (the η meson) becomes massive. Note that in this work we neglect the gluon sector.

At tree level, $\mathcal{L}_m$ is constructed to yield the leading-order results of the $1/N_c$ expansion of QCD. If one quantizes this theory with a cutoff, since the N_c dependence appears through the coupling constants, loop contributions will have an N_c dependence, which can be given by naive power counting. Such a power counting based on topological arguments shows that the meson n-point function is $O(N_c^{1-n/2-l})$, where l is the number of loops. The $1/N_c$ expansion of $\mathcal{L}_m$ is thus equivalent to its loop expansion. We conclude that perturbative sectors of $\mathcal{L}_m$ reproduce the qualitative features of QCD to all orders in $1/N_c$, and are exact in leading order.

We now introduce baryon fields into the Lagrangian. These may be viewed as a replacement of solitonic states of mesons, which are expected to emerge at low energy, as in the phenomenological Skyrme model.[15] Solitonic sectors are orthogonal to perturbative sectors, and overcounting can be avoided. The effective Lagrangian will then contain the baryonic kinetic term and a meson-baryon interaction term. Let us assume, for the time being, that such terms are local:

$$\mathcal{L}=\bar{\psi}(i\partial\!\!\!/-N_c m)\psi+\sqrt{N_c}g\,\bar{\psi}\psi\phi+\mathcal{L}_m\,, \qquad (2)$$

where ψ denotes the baryons collectively, and the Yukawa interaction term includes all possible bilinear covariants under the Lorentz transformation. The baryon ground states with various quantum numbers are degenerate in energy, as in the Skyrme model, with $J=I\leq N_c/2$. As in $\mathcal{L}_m$, through the explicit N_c dependence, $\mathcal{L}$ is constructed to yield, at tree level, the results of the $1/N_c$ expansion of QCD.

However, the loop contributions of $\mathcal{L}$ yield the N_c dependence that is different from the one expected by naive power counting, as can be explicitly confirmed in the case of one-loop diagrams.[18] For illustrative purposes, consider a meson propagator diagram with one baryon loop. While the naive counting gives $O(1/N_c)$ for this diagram, an explicit calculation yields $O(N_c)$, with an $O(N_c\ln(N_c))$ part that can be absorbed in the counter term. Since the meson propagator at tree level is of $O(1)$, the perturbative expansion cannot be expected to describe the $1/N_c$ expansion. Actually, the large-N_c limit of QCD yields a strong suppression of $O(N_c^{-N_c})$, since the baryon loop can be viewed as a two-point function of a quark bilinear with nested N_c-1 quark loops inside. These different N_c dependences show that $\mathcal{L}$ is not an effective Lagrangian of QCD beyond baryon-tree level, and all baryon-loop diagrams must be discarded if $\mathcal{L}$ is to be used.

The effective theory of QCD must also satisfy all relevant symmetry properties of QCD. Consider crossing symmetry: In the large-N_c limit, a baryon annihilation process is mapped under crossing into the corresponding baryon-scattering process at a large momentum transfer and vanishes as $\sim\exp(-\text{const}\times N_c)$.[13] The large-$N_c$ limit of QCD thus demands that the meson-baryon vertex depends on the momentum transfer.[19] Indeed, $\mathcal{L}$ of Eq. (2) lacks an important consequence of QCD, a baryon structure that appears as baryon sizes of $O(1)$ in the large-N_c limit.[13] The meson-baryon vertex form factor must then be suppressed beyond the momentum transfer $\sim O(1)$. However, could such a form factor emerge from radiative contributions in local field theories such as Eq. (2)? Our large-N_c expansion provides a simple negative proof to this often-discussed conjecture:[20] Radiative corrections to the vertex always include one or more baryon propagators. Since each propagator contributes $O(1/N_c)$, the vertex cannot swell up to $O(1)$.

The two preceding consequences show explicitly that the perturbative expansion of $\mathcal{L}$ is consistent with the $1/N_c$ expansion of both itself and QCD. That is, when we compute a perturbative result of $\mathcal{L}$ up to a certain order, the result does not generally correspond to what QCD yields in the large-N_c limit. The underlying physics is that the composite structure of the baryons cannot be neglected in nuclear phenomena, as previously emphasized.[3,21] The composite baryon structure will modify not only the vertex terms of $\mathcal{L}$ but also the baryon propagator. The resultant Lagrangian is nonlocal, in contrast to $\mathcal{L}_m$.

Consider the Yukawa interaction. The most general, nonlocal interaction that is Lorenz invariant and conserves momentum is (in momentum space)

$$L_y=\int d^4k_1d^4k_2f(k_1^2,k_2^2,k_1\cdot k_2)\bar{\psi}(k_1)M\psi(k_2)\phi(-k_1-k_2)\,, \qquad (3)$$

where M is an appropriate product of the γ matrices. According to our previous discussion of crossing symmetry, the general nonlocal coupling function f is of the form $f(|k_1+k_2|)$ and vanishes as $\exp(-aN_c)$ for the momentum $\sim O(N_c)$. Thus L_y, in fact, contains a form factor. Similarly, the general nonlocal form of the baryon kinetic term can be written as

$$L_k=\int d^4k\,\bar{\psi}(k)g(k)[k\!\!\!/+h(|k|)]\psi(-k)\,, \qquad (4)$$

where $g=1$ and $h=mN_c$ at $|k|=mN_c$, and counting rules at large N_c imply that $g(k)\sim\exp(\text{const}\times N_c)$ for either $|k_0-mN_c|$ or $|\mathbf{k}|\sim O(N_c)$.[13] The baryon propagator generated by L_k thus suppresses baryon loops in accordance with large-N_c QCD. (Note that the conventional vertex form factors do not suppress baryon loops.)

Since terms of higher-mass dimensions are suppressed as in $\mathcal{L}_m$, the combination of L_k and L_Y serves as an effective Lagrangian of QCD: As used as a field theory, it yields the *leading- and subleading-order* results consistent with the large-N_c QCD. This is our major result.

The nonlocal forms of L_k and L_Y are technically complicated and cumbersome to use. We have found, however, that once baryon loops are omitted, $\mathcal{L}$ of Eq. (2) with form factors yields, for a given hadronic diagram, the same *leading-order* result as both the corresponding QCD diagram and the nonlocal Lagrangian. This procedure of using $\mathcal{L}$ does not guarantee consistency with the large-N_c QCD in the subleading order, but such a defect perhaps can be compensated by properly adjusting parameters in the theory. This procedure, intrinsically phenomenological, can serve as a pragmatic alternative to the use of the nonlocal Lagrangian. The procedure is the same as the traditional approaches in nuclear physics, including the relativistic many-body theories[1,2] used at the baryon-tree level. Some recent phenomenological works indeed show unphysical consequences emerging from baryon loops.[7]

Let us discuss some examples to illustrate that our pragmatic procedure yields results consistent with the $1/N_c$ expansion of QCD. Consider the meson-baryon interaction: The proper vertex (self-energy) diagram consists of two Yukawa couplings with a baryon propagator between them. By inspection of $\mathcal{L}$, i.e., by a naive N_c counting, we see that each coupling contributes as $O(\sqrt{N_c})$ and each baryon propagator as $O(1/N_c)$. Thus, this hadronic diagram is of $O(1)$, just as corresponding QCD diagrams (describing the same process) contribute in the large-N_c limit.[13] In turn, the hadronic diagram yields a scattering amplitude and cross section of $O(1)$. Actually, this step involves a sublety. External baryon lines force the baryon propagator to contribute $O(1)$, and external meson lines contribute in two ways: crossed and uncrossed. Each amplitude diagram then contributes $O(N_c)$, but the sum of the cross and uncrossed diagrams yields $O(1)$ after cancellation.[22] In order to avoid such a sublety, we restrict our discussion to *proper vertices*, except where crossing properties are examined.

Consider next the two-baryon interaction. At tree level, one-meson exchanges consist of two Yukawa couplings and a meson propagator. Since the meson propagator contributes as $O(1)$, the diagrams are of $O(N_c)$ and again correspond to QCD diagrams in the large-N_c limit.[13] Beyond the tree level (but still at the baryon-tree level), all of the naive N_c counting and the explicit loop calculations with or without cutoffs yield two-meson exchange diagrams of $O(1)$. An extension of Witten's method[13] tells us that the corresponding large-N_c QCD diagrams are those of exchanges of two quark-antiquark pairs and give the same order. Note that the QCD diagrams must be one-particle irreducible and must contain baryons in the color-singlet state. In the same way, m-meson exchange diagrams of two-baryon interactions are found to be of $O(N_c^{2-m})$.

These strong two-baryon interactions generally agree with the descriptions based on the relativistic many-body theories. As in these theories, our NN interactions of $O(N_c)$ must cancel to yield the small nuclear binding energy, so as to agree with the large nuclear sizes. We also find that three- and four-baryon forces contribute to $O(N_c)$, as strong as those by conventional meson-exchange calculations.[23] As in these calculations, substantial cancellations must then exist among various diagrams (but without baryon loops).

Our pragmatic procedure agrees with that of Brodsky, obtained in the perturbative regime of the large momentum-transfer phenomena.[21] In addition, his Lagrangian includes the suppression of the $N\bar{N}$ pair terms. In the low energy, it is also desirable to suppress the so-called πN Z graph with the pseudoscalar coupling. Contrary to his high-energy case, this graph cannot be suppressed by a conventional vertex form factor. Our effective nonlocal Lagrangian suppresses it by the L_k, and we expect that the generation of N_c quark-antiquark pairs is indeed strongly suppressed in the corresponding QCD diagram. This problem, however, may be a special case in that the suppression is a direct consequence of QCD symmetry, such as chiral symmetry, as it has been demonstrated by the use of σ models. Since dropping a tree-level diagram by hand generally violates unitarity, it must be treated with care. This problem is under further study.

How far can we apply our effective Lagrangian? While it should be applicable for the momentum of $O(1)$, it may be applicable even to $O(N_c)$, once the baryon form factors are included. Indeed, recent deuteron photodisintegration data deviate from a perturbative QCD estimate but agree with meson-exchange calculation below 1 GeV.[24]

A general note. The baryon loops discussed in this work are vacuum fluctuations appearing in field theories, but are not particle-hole states appearing in many-body theories. Since no distinctly different QCD diagrams correspond to such states, the latter requires no special treatment. The actual amount of their contributions would be somewhat altered, however, by the introduction of the form factors due to the relatively small Fermi momentum of about 720 MeV/c.

After this work was submitted for publication, we were informed of work by T. Cohen,[25] who has noted baryon-loop suppression (but restricted to the one-nucleon-loop level) in the large-N_c limit of QCD.

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