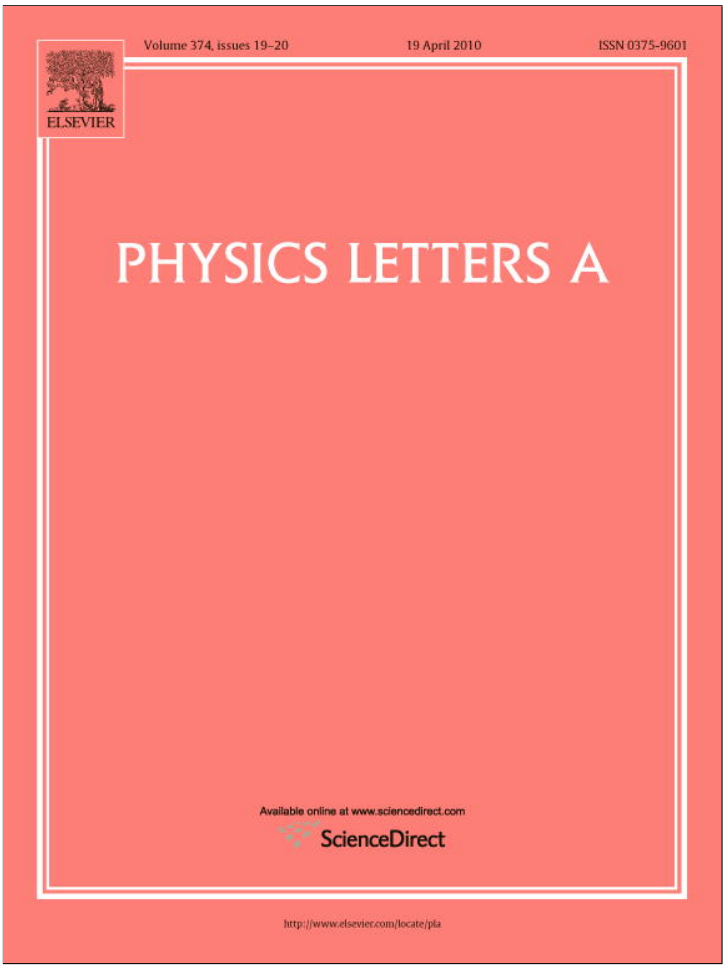

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Multistability and localization in coupled nonlinear split–ring resonators

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ABSTRACT

We study the dynamics of a pair of nonlinear split–ring resonators (a 'metadimer') excited by an alternating magnetic field and coupled magnetically. Linear metadimers of this kind have been recently used as the elementary components for three-dimensional metamaterials or 'stereometamaterials' by N. Liu et al. (2009) [9]. We demonstrate that nonlinearity offers more possibilities with respect to real-time tunability and a multiplicity of states which can be reached by varying the external field. Moreover, we demonstrate almost total localization of the energy in one of the resonators in a broad range of parameters.

1. Introduction

Metamaterials are artificially structured composites which exhibit electromagnetic properties not available in naturally occurring materials. Such structures are largely based on subwavelength resonant 'particles' referred to as split–ring resonators (SRRs). Scaling down the size of the SRRs allows to realize metamaterials up to Terahertz and optical frequencies [1–3]. However, for exploiting the great potential of metamaterials for applications, it is desirable to change their effective parameters in real time, i.e., to achieve real-time tunability. This is also motivated the construction of nonlinear SRRs, whose structure is very well suited for enhancing nonlinear phenomena [4]. An effective way of constructing an easily controllable nonlinear SRR is the insertion of a nonlinear electronic component into its slit [5–7]. The arrangement of a large number of nonlinear SRRs into a periodic lattice results in a nonlinear magnetic metamaterial which is tunable by varying the power of the applied field [8].

It was recently suggested that dimers, comprised of two spatially separated SRRs, can be used as elemental units for the construction of three dimensional metamaterials (stereometamaterials') [9]. The close proximity of the SRRs in the dimer results in relatively strong coupling between them. A metamaterial comprised of a large number of such metadimers can be utilized as a tunable optically active medium [10]. Moreover, if one or both SRRs in the metadimer become nonlinear, the metamaterial itself acquires nonlinear properties, essential for real-time tunability. It is therefore of great importance to investigate the nonlinear properties of the elementary unit of such a dimer-based metamaterial, i.e., of the nonlinear metadimer, which can be modeled as system of two coupled nonlinear oscillators.

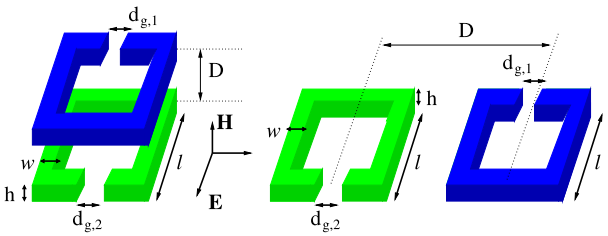

Fig. 1. (Color online.) Schematic of a broad-side (left panel) and narrow-side (right panel) asymmetric metadimer.

2. Model metadimer

We consider an asymmetric metadimer comprised of two SRRs which have slightly different slit widths as in Fig. 1, $d_{g,1}$ and $d_{g,2}$, which differentiates their linear capacitances C_1 and C_2, respectively [11,12]. That in turn differentiates the linear resonance frequencies of the SRRs through the relation $\omega_i \simeq 1/\sqrt{LC_i}$ $(i = 1, 2)$, while the changes in the inductance L and the Ohmic resistance R of the SRRs are of higher order. The relative orientation of the SRRs in the metadimer can be such that the SRRs are either narrow-side coupled or broad-side coupled (right and left panel of Fig. 1, respectively). The nature of the interaction between the SRRs (elec-

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tric or magnetic or both) is determined by the twist angle of the slit of one of the SRRs around the **H** field with respect to the other. In both geometries shown in Fig. 1, that angle is 180° so that the distance between the slits is much larger than their widths. Then, the interaction between the SRRs is predominantly magnetic and the electric dipole–dipole interactions can be neglected [13,14].

The metadimer is placed in an alternating electromagnetic field with the polarization shown in Fig. 1, so that only its magnetic component is capable of exciting induced currents in the SRR rings. In the equivalent circuit picture the metadimer is regarded as a pair of periodically driven, nonlinear resistor–inductor–capacitor (RLC) oscillators coupled magnetically through their mutual inductance M, driven by identical voltage sources. Consider a metadimer with the geometry shown in the left panel of Fig. 1, for which the magnetic coupling between the SRRs is relatively strong. Then, the equations describing the dynamics of the (normalized) charge q_i, accumulated across the slit of the i-th oscillator, read [12]

$$\ddot{q_1} + \lambda_M \ddot{q_2} + \gamma \dot{q_1} + (\partial u_1/\partial q_1) = \varepsilon_0 \sin(\Omega\tau), \tag{1}$$

$$\ddot{q_2} + \lambda_M \ddot{q_1} + \gamma \dot{q_2} + (\partial u_2/\partial q_2) = \varepsilon_0 \sin(\Omega\tau), \tag{2}$$

where $\lambda_M = M/L$ is the magnetic interaction strength, $\omega_0 = \sqrt{\omega_1\omega_2}$ a characteristic frequency, $\varepsilon = \varepsilon_0 \sin(\Omega t)$ is the induced electromotive (emf) force excited in each SRR, $\gamma = R\sqrt{\sqrt{C_1C_2}/L}$ is the damping constant, the overdots denote derivation with respect to the normalized temporal variable τ, and

$$u_1 = \frac{\delta}{2} q_1^2 \left(1 - \frac{\delta^2}{2}\chi q_1^2\right), \qquad u_2 = \frac{1}{2\delta} q_2^2 \left(1 - \frac{1}{2\delta^2}\chi q_2^2\right), \tag{3}$$

with $\delta \equiv \omega_1/\omega_2$ being the resonance frequency mismatch (RFM) parameter, which quantifies the asymmetry of the metadimer. The average energy, E_{tot}, of the metadimer can be obtained from the time-average in one period of the Hamiltonian

$$H = \frac{1}{2(1-\lambda_M^2)}\left(p_1^2 + p_2^2 - 2\lambda_M p_1 p_2\right) + u_1 + u_2, \tag{4}$$

where $p_1 = \dot{q}_1 + \lambda_M \dot{q}_2$ and $p_2 = \dot{q}_2 + \lambda_M \dot{q}_1$, with p_i and q_i calculated from Eqs. (1) and (2). For normalizing the earlier equations, we have scaled charge, voltage, time, and frequency by Q_c, U_c, ω_0^{-1}, and ω_0, respectively, where $Q_c = \sqrt{C_1C_2}U_c$, $U_c = \sqrt{d_{g,1}d_{g,2}}E_c$, with E_c a characteristic electric field amplitude.

3. Multistability and localization

The rich dynamical behavior of the nonlinear metadimer can be observed in Fig. 2 where a typical bifurcation diagram of the currents i_1 and i_2 is shown as a function of the driving frequency Ω. That diagram can be divided into three regions; the region at left, where two stable periodic solutions coexist for a wide Ω interval, the region at right where there is a single stable solution (except in a narrow Ω interval), and the chaotic region in between separating the previous two. Multistability exists from $\Omega = 0.6$ to 0.87, where there are two stable states with very different energies; the high and low energy state with $E_{tot} = 1.43$ and $E_{tot} = 0.014$, respectively. Importantly, those two states differ considerably in the distribution of E_{tot} in the two oscillators. We define the energy fractions in oscillators 1 and 2 as $e_1 = E_1/E_{tot}$ and $e_2 = E_2/E_{tot}$, respectively, where E_1 and E_2 are their energies. The high energy state has $e_1 = 0.993$ and $e_2 = 0.007$ while the low energy state has $e_1 = 0.57$ and $e_2 = 0.43$. Thus, in the former case, almost all the energy is localized in the first of the oscillators.

The total energy E_{tot} of the two states and the energy fractions e_1 and e_2 of the two oscillators as a function of Ω are shown in Figs. 3 and 4, respectively. In that specific case, extreme localization occurs in a rather wide Ω interval, at least from 0.6 to 0.87. If

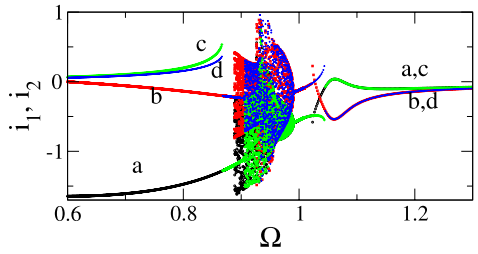

Fig. 2. (Color online.) Bifurcation diagram of the currents i_1 and i_2 as a function of the driving frequency Ω, for $\delta = 0.95$, $\gamma = 0.01$, $\chi = +1/6$, $\lambda = 0.12$, and $\varepsilon_0 = 0.06$. Decreasing Ω: i_1 and i_2 correspond to black and red symbols (a and b), respectively; increasing Ω: i_1 and i_2 correspond to green and blue symbols (c and d), respectively.

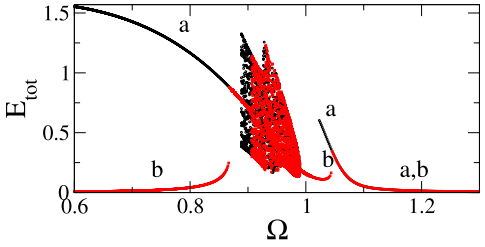

Fig. 3. (Color online.) Total energy of the metadimer E_{tot} as a function of the driving frequency Ω, for the parameters in Fig. 2, and Ω decreasing from 1.3 (black symbols – a); Ω increasing from 0.6 (red symbols – b).

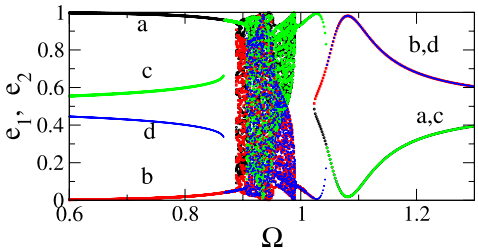

Fig. 4. (Color online.) Energy fractions e_1 and e_2 as a function of the driving frequency Ω for the parameters in Fig. 2. Decreasing Ω: e_1 and e_2 correspond to black and red symbols (a and b), respectively; increasing Ω: e_1 and e_2 correspond to green and blue symbols (c and d), respectively.

the metadimer is initially driven with low frequency it settles into the low energy state. As the frequency increases, it passes through the point where that state becomes unstable (at $\Omega = 0.865$), and the metadimer suddenly switches to the high energy state. We also note that for $\Omega = 0.885$ to 0.895 a stable periodic state coexist with a chaotic state (blue in red and green in black).

In the right-most region of Fig. 2, there is also a narrow Ω interval (from $\Omega = 1.02$ to 1.05) where multistability occurs. Outside that interval, i.e., from $\Omega = 1.05$ to 1.3 the dimer state has very low energy (see Fig. 3). However, there are two specific values of Ω, i.e., at $\Omega = 1.08$ and 1.025, where E_{tot} is highly localized. At those points the energy fractions are $e_1 = 0.02$, $e_2 = 0.98$ and $e_1 = 0.99$, $e_2 = 0.01$, respectively. Notice that the state at $\Omega = 1.025$ lies in the region of bistability in this part of the diagram, which has the lowest energy, while it also exhibits localization. The corresponding higher energy state at this frequency has $e_1 = 0.46$ and $e_2 = 0.54$, so that E_{tot} is almost equally distributed in the two oscillators (see also Fig. 4).

The currents i_1 and i_2 for $\Omega = 0.7$ ($T = 8.98$), which lies in the region where both multistability and localization occur, are shown in Fig. 5 as a function of τ, both for the high (Fig. 5a) and the low (Fig. 5b) energy states. In the same figures we also plot $\cos(\Omega\tau)$ which is directly proportional to the applied magnetic

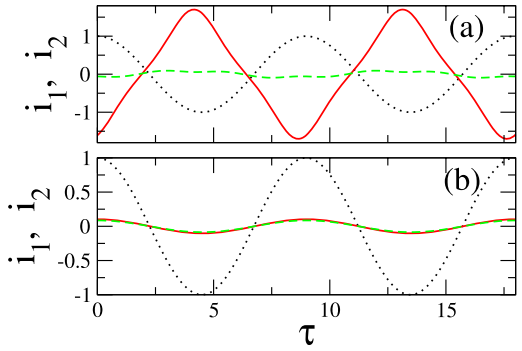

Fig. 5. (Color online.) Temporal evolution of the currents at $\Omega = 0.7$, for (a) the high energy state; (b) the low energy state. The other parameters are as in Fig. 2.

field. The relative phase difference between the applied magnetic field and the current in each SRR, which is directly proportional to its magnetic moment, determines the response of the metadimer to that field. We observe that the currents of the metadimer in the high energy state have a relative phase difference of almost 180° with respect to the applied magnetic field, indicating a *diamagnetic* magnetic response. To the contrary, the currents of the metadimer in the low energy state are in phase with the applied magnetic field, indicating a *paramagnetic* response.

4. Conclusions

In conclusion, we have revealed a physical mechanism of the intrinsic localization of energy in nonlinear magnetic metamaterials in the study of the dynamics of coupled split-ring resonators. In particular, we have found that extreme localization of energy may occur in a slightly asymmetric nonlinear metadimer, which also exhibits multistability of states and chaos for a rather wide range of parameters. For a symmetric metadimer (with $\delta = 1$ and other parameters as in Fig. 2), no localized states and chaos have been observed, while multistability still occurs. Thus, it seems that a slight asymmetry is required for localization to occur.

We have found that, in the multistability regions, there appear two stable states which differ considerably in their energies. The magnetic response of the metadimer is either paramagnetic or diamagnetic, depending on the energy of the state of the metadimer (low and high, respectively). The magnetic response of a magnetic metamaterial comprised of such metadimers is determined by averaging the response of the individual metadimers. When the metadimers are in a high energy state, they may respond extremely diamagnetically to an applied field. Then, the response of the magnetic metamaterial would be described macroscopically by a relatively large and negative magnetic permeability parameter.

We believe that this study may be useful for realizing strongly nonlinear effects and energy localization in metamaterials comprised of a large number of nonlinear split–ring resonators where strongly localized states in the form of discrete breathers [15–18], as well as other types of localized excitations such as domain walls and envelope solitons [19,20] were predicted theoretically.

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